Reshoring Policies in the Intangible Economy*

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Abstract

Incorporating intangible scalability into a two-country general equilibrium model with firm heterogeneity and endogenous entry, this paper analyzes the macroeconomic effects of reshoring policies—lump-sum subsidies, tax credits, and tariffs—on intangible-intensive offshoring firms. All three policies generate medium-run consumption expansions despite eroding offshore cost advantages. Tax credits maximize reshoring value-added by directing support toward high-productivity firms, while lump-sum subsidies produce the largest transitional welfare gains through more pronounced wage growth. While tariffs boost consumption through revenue rebates, they persistently suppress output by contracting foreign demand and offshore value-added. The calibrated Ramsey-optimal policy mix—a 37.0% lump-sum subsidy combined with a 9.6% tariff—enhances welfare by 2.19% relative to the no-policy benchmark. Ultimately, intangible scalability dictates the efficacy of reshoring policies, generating welfare gains that offset the resource misallocation costs of global disintegration.

JEL Codes: L5, F13, E22, F12, F41

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1 Introduction

Recent U.S. reshoring policies, as manifested in the CHIPS and Science Act and Executive Order 14017, explicitly target the return of intangible-intensive manufacturing—sectors defined by heavy reliance on R&D, software, and intellectual property, including semiconductors, battery manufacturing, and aerospace[1]—to revitalize the domestic economy. By ensuring the retention of intellectual property and R&D-heavy production within national borders, these initiatives aim to consolidate global technological leadership and secure high-value-added supply chains.[2] Therefore, reshoring in advanced economies today functions as a strategic industrial policy designed to preserve national innovation capabilities.

This trend directly contradicts the traditional principles of multilateralism, which emphasize the efficiency of resource allocation through global integration. According to Melitz (2003), global integration generates aggregate productivity gains by reallocating resources to the most productive firms. Consequently, reshoring policies—by incentivizing a return to domestic production at the expense of foreign cost advantages—inevitably result in resource misallocation and macroeconomic inefficiencies.

However, the presence of intangible capital raises new questions regarding the logic of inefficiency caused by such global disintegration. Unlike physical capital, intangible capital is characterized by scalability, which drives marginal costs near-zero as production scales (Haskel and Westlake, 2017; Hsieh and Rossi-Hansberg, 2023). This unique feature induces a fundamental shift in a firm's profitability structure when intangibles are relocated internationally. This perspective is supported by recent views (e.g., Goldberg, Juhász, Lane, Lo Forte, and Thurk, 2024) that industrial policies targeting the domestic resurgence of intangible-intensive sectors, such as semiconductors, can drive economic growth and productivity gains by lowering marginal costs and realizing economies of scale. In essence, reshoring policies that incentivize domestic intangible investment open the possibility for efficiency gains that may offset the costs of resource misallocation.

Despite this possibility, the literature has yet to provide a rigorous general equilibrium analysis of how reshoring policies alter the offshoring decisions of intangible-intensive firms and, consequently, shape aggregate macroeconomic performance. Existing protectionism literature has predominantly focused on resource misallocation costs without considering the efficiency gains induced by intangibles, nor has it accounted for how the unique cost

[1]Intangible capital is defined as Intellectual Property Products under the System of National Accounts (SNA), encompassing R&D, computer software and databases, and licensing agreements. Representative intangible-intensive industries include Semiconductors (NAICS 3344), Battery Manufacturing (NAICS 3359), Computer Equipment (NAICS 3341), and Aerospace (NAICS 3364).

[2]For instance, the Presidential Memorandum on Section 301 (2018) states that imposing strategic tariffs on R&D-intensive goods serves to protect the domestic production of intangible-intensive goods and correct unsustainable trade imbalances.

structure of intangibles influences firms' production location decisions. While research on industrial policies in intangible-intensive industries has grown, it remains confined to partial equilibrium approaches that abstract away from the endogenous firm dynamics central to offshoring. This paper fills that gap by asking: How do reshoring policies affect offshoring decisions and aggregate macroeconomic outcomes in the presence of intangible capital?

To address this question, I incorporate intangible scalability as the central mechanism in a two-country offshoring general equilibrium model with heterogeneous firms and endogenous entry and exit. The model builds on Lee (2025)—which integrates the intangible investment mechanism of De Ridder (2024) into an offshoring framework—by treating intangibles as a fixed cost that reduces marginal production costs for heterogeneous firms across Home and Foreign economies. Consistent with Ghironi and Melitz (2005, GM05), the model features endogenous firm entry, heterogeneous labor productivity, and endogenous export decisions. Following Zlate (2016, Zlate16), I assume an asymmetric offshoring framework where only Home firms establish offshore plants in Foreign to exploit wage differentials and reship output to serve the domestic market. Within this general equilibrium environment, firms endogenously decide whether to invest in scalable intangible capital and determine their optimal production location on a period-by-period basis.

Home firms choose among three production modes: (i) offshoring with intangibles, (ii) domestic production with intangibles, and (iii) domestic production without intangibles. In this framework, offshoring firms are required to invest in intangibles—incurring additional fixed costs while benefiting from lower marginal costs.[3] Domestic firms are further differentiated by intangible investment status: those with intangibles bear the associated fixed cost but enjoy lower marginal costs, while those without intangibles face neither. More productive firms achieve greater marginal cost reductions[4], so firms sort across modes according to endogenous productivity cutoffs—with the most productive firms selecting offshoring, followed by domestic production with intangibles, and then domestic production without intangibles.

To ensure policy relevance, I analyze the three most prevalent reshoring instruments documented in the New Industrial Policy Observatory (NIPO): lump-sum intangible subsidies, tax credit subsidies, and import tariffs. Lump-sum subsidies reduce the fixed costs of domestic intangible investment, while tax credits lower variable production costs for firms engaging in domestic intangible investment. Together, these subsidy instruments represent more than

[3]This is consistent with empirical evidence showing that offshoring firms exhibit greater R&D intensity and technology investment than non-offshoring firms (Bøler, Moxnes, and Ulltveit-Moe, 2015; Dachs, Ebersberger, Kinkel, and Som, 2015; Fritsch and Gorg, 2015), and that German multinationals show higher innovation rates and offshore more R&D (Gumpert, Manova, Rujan, and Schnitzer, 2024). Furthermore, Compustat Historical Segment data indicate that offshoring US manufacturers invest more heavily in intangibles than their non-offshoring counterparts (see Appendix A, Stylized Fact (iv)).

[4]For simplicity, intangible fixed costs are uniform within each production mode, while the resulting marginal cost reductions are heterogeneous across firms.

half of all industrial policies deployed in advanced economies. Import tariffs, exemplified by the strategic use of Section 301 tariffs—which contributed to Tesla's decision to reshore its Shanghai-based battery production to Nevada—erode the profitability of foreign production and act as an indirect incentive to reshore.

This study examines whether the intangibles reshoring gain induced by reshoring policies can offset the resource misallocation costs dynamically resulting from global disintegration, based on changes in macroeconomic variables. Furthermore, I aim to investigate how each policy instrument reshapes firms' microeconomic offshoring decisions and, consequently, its implications for aggregate macroeconomic welfare.

The key findings are as follows. First, reshoring policies in an intangible-intensive economy generate medium-run expansions in consumption, despite eroding the cost advantages of offshore production. Both lump-sum subsidies and tax credits encourage domestic production, thereby stimulating domestic labor demand and wages, which in turn expands both output and consumption. Tariffs bolster consumption through revenue transfers to households, though they contract Home output by reducing Foreign exports and shrinking offshore value added.

Second, tax credits generate the highest reshoring value-added by aligning incentives with productivity. Unlike lump-sum subsidies, tax credits scale with output, meaning the most efficient firms receive the greatest support. This mechanism encourages high-productivity firms—which achieve greater marginal cost reductions—to reshore, maximizing value creation. Lump-sum subsidies, provided regardless of productivity, attract relatively less productive firms and yield lower aggregate value-added. Tariffs, which raise foreign costs without directly improving domestic profitability, generate the least reshoring value-added.

Third, all reshoring policies targeting intangible-intensive firms induce transitional welfare gains through intangible scalability. Both types of subsidies promote reshoring, which bolsters domestic labor demand and drives up wages, thereby expanding consumption and welfare. Notably, lump-sum subsidies generate the largest welfare gains; by attracting relatively less productive firms compared to tax credits, they trigger a more pronounced increase in domestic wages, generating the largest expansion in household labor income during the transition. Tax credits yield the second-largest welfare gains, while tariffs produce the smallest. Tariffs boost consumption primarily because revenues are rebated to households via lump-sum transfers in the absence of foreign retaliation.

Finally, the calibrated Ramsey-optimal reshoring policy mix—comprising a 37.0% lump-sum subsidy on reshoring firms' profits and a 9.6% tariff—enhances welfare by 2.19% relative to the no-policy benchmark. The optimality of combining subsidies with tariffs reflects a complementarity: subsidies drive wage growth and domestic expansion, while tariffs penalize foreign-produced varieties and redirect demand toward domestic alternatives. These findings imply that reshoring policies targeting intangible-intensive industries, despite eroding the cost advantages of offshore production, can ultimately generate welfare gains by incentivizing domestic intangible investment.

This paper then provides empirical evidence supporting the economic forces and assumptions highlighted by the model. First, reshoring subsidies significantly increase value added (VA) in intangible-intensive manufacturing industries, whereas tariffs significantly reduce VA in these sectors. Second, reshoring subsidies drive a significant increase in real personal consumption expenditures (PCE) within those same industries. Third, tax credits show a stronger tendency to incentivize the reshoring of high-productivity offshore firms compared to other policies. Collectively, these empirical regularities are consistent with the model's mechanisms and suggest that reshoring policies targeting intangible-intensive industries have the potential to stimulate the domestic economy rather than merely generating inefficiencies.

The primary contribution of this study lies in providing a new theoretical framework to evaluate reshoring policies and their welfare implications by integrating the scalability of intangible capital into an offshoring general equilibrium model with endogenous firm entry and exit. Barattieri, Cacciatore, and Ghironi (2021) demonstrate that tariffs are ineffective as a macroeconomic stimulus within a general equilibrium framework with endogenous firm entry and exit; this study shows how endogenous economies of scale driven by intangibles can overturn that result in the medium run. Goldberg, Juhász, Lane, Lo Forte, and Thurk (2024) identify that industrial subsidies can revitalize the economy through learning-by-doing in a partial equilibrium framework; this study extends the mechanism to a general equilibrium setting to evaluate aggregate resource allocation and welfare. Most distinctively, this paper provides what is, to my knowledge, the first general equilibrium analysis of reshoring policies within an offshoring framework—directly speaking to how such policies restructure global value chains and shape national competitiveness and welfare.

Outline The rest of the paper is organized as follows. Section 2 develops the benchmark model. Sections 3 and 4 detail the reshoring policies and calibration, respectively. Section 5 presents the calibration results, and Section 6 provides the corresponding empirical evidence. Section 7 conducts the welfare analysis, and Section 8 concludes the paper. The Appendix provides analytical derivations, empirical support for model assumptions, and event-study findings.

2 The Model

The model is composed of two economies, Home and Foreign, each with a representative household and a continuum of monopolistically competitive firms with heterogeneous labor productivity producing differentiated varieties. Home firms can produce domestically or offshore for their domestic market, while Foreign firms produce only domestically due to

Home's higher labor costs. Additionally, a subset of firms from both countries produce domestically to export.

This section details the representative household's and firms' problems in the Home economy under financial autarky. For simplicity, labor is the sole production input. Appendix C provides full model summaries.

2.1 Household Problem

Households maximize their expected intertemporal utility, which depends on their consumption level (C_t). The expected intertemporal utility function is:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma}}{1-\gamma} \tag{1}$$

The parameter $\beta \in (0,1)$ is a discount factor, and $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution. The consumption basket (C_t) is defined over a continuum of goods Ω, where the elasticity of substitution across goods is constant, $\theta > 1$.

The household purchases the consumption basket C_t, including varieties produced domestically by Home firms without intangible investment ($\omega \in \Omega_t^{NI}$), with intangible investment ($\omega \in \Omega_t^{I}$), varieties produced offshore ($\omega \in \Omega_t^{V}$), and varieties produced by Foreign firms and imported by Home ($\omega \in \Omega_t^{F}$), all with the symmetric elasticity of substitution $\theta > 1$:

$$C_t = \left[\underbrace{\int_{z_{min}}^{z_{D,t}^I} y_{D,t}^{NI}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega\in\Omega_t^{NI}} + \underbrace{\int_{z_{D,t}^I}^{z_{V,t}} y_{D,t}^{I}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega\in\Omega_t^{I}} + \underbrace{\int_{z_{V,t}}^{\infty} y_{V,t}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega\in\Omega_t^{V}} + \underbrace{\int_{z_{X,t}^*}^{\infty} y_{X,t}^{*}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega\in\Omega_t^{F}} \right]^{\frac{\theta}{\theta-1}} \tag{2}$$

$[z_{min}, \infty)$ is the support interval for the idiosyncratic productivity of Home firms. Firms exceeding the endogenous threshold $z_{D,t}^I$ invest in intangible capital for the domestic market, while those above $z_{V,t}$ offshore production. The composition of the consumption basket changes over time since the number of firms is time-variant and firms re-optimize their intangible investment, offshoring and exporting strategies every period. $p_t(\omega)$ denotes the home currency price of a good $\omega \in \Omega_t$, and the consumption-based price index for the home economy is $P_t = (\int_{\omega\in\Omega} p_t(\omega)^{1-\theta} d\omega)^{1/(1-\theta)}$. The household's demand for each individual good ω is $c_t(\omega) = (p_t(\omega)/P_t)^{-\theta} C_t = \rho_t(\omega)^{-\theta} C_t$, where $\rho_t(\omega)$ is the real price of each variety and $\omega \in \Omega_t^{NI} \cup \Omega_t^{I} \cup \Omega_t^{V} \cup \Omega_t^{F}$.

2.2 Firm entry

In each period, an unbounded mass of potential entrants exists in both Home and Foreign. Following GM05, firm entry occurs each period in both countries and requires paying a sunk cost $f_{E,t}$, measured in effective labor units of the respective country. Upon entry, each firm draws its productivity level z from a common distribution $G(z)$ supported over the interval $[z_{min}, \infty)$, which remains fixed thereafter.

New entrants are forward-looking and calculate the expected stream of future profits $\tilde{d}_t$ in every period. Entrants at time t only start producing at time $t+1$, introducing a one-period time-to-build lag in the model. The exogenous exit shock occurs at the end of the time period with the probability δ. Prospective home entrants in period t compute their expected post-entry value $\tilde{v}_t$ given by the present discounted value of their expected stream of profit $\{\tilde{d}_s\}_{s=t+1}^{\infty}$:

$$\tilde{v}_t(z) = E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_s}{C_t}\right)^{-\gamma} \tilde{d}_s(z) \tag{3}$$

Thus, $N_{E,t}$ new firms are created every period t and $(1-\delta)N_{E,t}$ firms start producing at t+1. The law of motion for the number of active firms is: $N_{t+1} = (1-\delta)(N_t + N_{E,t})$. The equation (3) also represents the average value of incumbent firms after production has occurred. Entry occurs until the average firm value is equalized with the entry cost, leading to the free entry condition $\tilde{v}_t = \frac{w_t}{Z_t} f_{E,t}$.

2.3 Firms' choice of markets and production strategies

Each period, the N_t active firms endogenously choose their production locations and whether to invest in intangible capital. The setup is as follows: (1) To serve the home market, domestic firms can choose three types of production methods: Producing domestically with intangible investment, producing domestically without intangible investment or producing in an offshore plant with intangible investment. Intangible investment offers the advantage of reducing marginal costs of production but incurs intangible fixed costs each period. Similarly, offshoring provides the advantage of lower wage costs but entails offshoring fixed and trade costs per period. It is essential to note that all firms, regardless of their choices regarding intangible investment or offshore production, are exclusively focused on serving their domestic market and do not engage in foreign market activities. (2) A subset of firms in each economy exports to serve foreign markets. This study focuses on intangible-intensive manufacturing firms. Given the empirical prevalence of vertical foreign direct investment (FDI) in manufacturing, this paper abstracts from outsourcing driven by horizontal FDI aimed at capturing foreign market share. Consequently, I assume that firms serving foreign markets produce domestically and export, subject to a fixed cost, as in GM05.

2.3.1 Intangible investment decision

This paper models intangible capital investment as recurring fixed costs that reduce marginal production costs, reflecting the scalable nature of intangibles. Scalability refers to the minimal marginal costs of reproducing intangible assets, which alters firms' cost structures by shifting costs from marginal to fixed components (De Ridder24). For instance, incorporating intangible capital such as R&D or software, with negligible replication costs into production, raises fixed costs, such as recurring expenses for software licensing fees or R&D wages, while keeping marginal costs low. As a result, firms' production decisions increasingly hinge on fixed costs. Moreover, the impact of this investment depends on firms' heterogeneous productivity: more productive firms achieve larger marginal cost reductions from the same fixed investment (Veugelers et al., 2020)[5].

The following analysis will explore the cost dynamics of firms with and without intangible investments. If a firm with heterogeneous productivity z does not undertake intangible capital investment, its marginal cost is equivalent to wages measured in units of effective labor.

$$mc_t(z) = \frac{w_t}{Z_t z} \tag{4}$$

If a firm with heterogeneous productivity z invests in intangible capital, its marginal costs will decrease by a factor of ψ_t^ξ. ψ_t denotes the fraction by which marginal costs are reduced, and ξ represents the cost elasticity of intangibles.

$$mc_t(z) = \psi_t^\xi \frac{w_t}{Z_t z} \quad \text{where } \psi_t < 1 \tag{5}$$

However, firms must incur a fixed cost, measured in units of consumption, in order to engage in intangible capital investment.

$$\phi_t \frac{w_t}{Z_t}(\psi_t^{-\xi} - 1) \tag{6}$$

Here, ϕ_t represents the share of intangible fixed costs in total fixed costs. Countries with lower ϕ_t have more efficient firms, reflecting higher intangible capital efficiency, as they achieve greater marginal cost reductions for the same fixed costs.

Firms invest in intangible capital only if the resulting profit $d_{D,t}^I(z)$ meets or exceeds the profit without it, $d_{D,t}^{NI}(z)$ (i.e., $d_{D,t}^I(z) \geq d_{D,t}^{NI}(z)$). Since intangible investment entails fixed costs, only firms with sufficiently high productivity can justify the investment.

[5]The scalability—where marginal costs decline with output—can also be confirmed using Compustat Segment Historical data. Related analysis is provided in Appendix A, Figure 10.

2.3.2 The timing of production and intangible investment decisions

After entry, firms draw their idiosyncratic productivity z from the common distribution $G(z)$. They then choose their production location and whether to invest in intangible capital based on their productivity. Without intangible investment, marginal costs are $\frac{w_t}{Z_t z}$; with investment, firms pay a fixed cost to reduce marginal costs to $\psi_t^{\xi} \frac{w_t}{Z_t z}$. Highly productive firms may both invest in intangibles and offshore production. Firms make these decisions simultaneously and then set prices and quantities to maximize profits. Figure 1 illustrates the timing of these decisions.

Since the decision to undertake intangible capital investment is made after the realization of productivity, firms cannot determine whether to make such investments prior to entering the market. Therefore, intangible capital investment decisions are made on a period-by-period basis according to the idiosyncratic productivity realized after entry. This setup can be considered a more realistic representation of the recurrent costs that firms must incur to maintain R&D, software, and information technology investments.

Since firms observe productivity only after entry, they cannot decide on intangible investment beforehand. Instead, they choose investment each period based on realized idiosyncratic productivity. This framework better captures the recurring costs firms incur to sustain R&D, software, and IT investments.

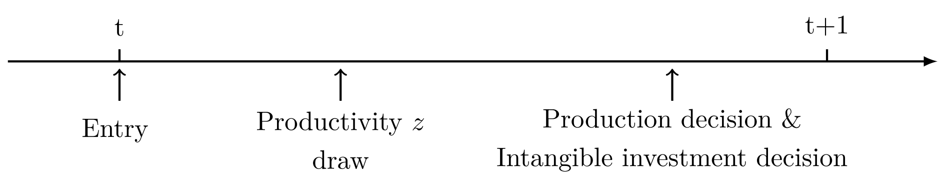

Figure 1: The timing of production and intangible investment decisions

2.3.3 Production decision

A home firm with idiosyncratic productivity z chooses between two production strategies each period to serve the home market: (a) Produce domestically, with output $y_{D,t}(z) = Z_t z l_t(z)$, as a function of aggregate productivity Z_t, the firm-specific labor productivity z, and domestic labor $l_t(z)$; or (b) Produce offshore, with output $y_{V,t}(z) = Z_t^* z l_t^*$, using foreign labor $l_t^*(z)$, subject to aggregate foreign productivity Z^*, while retaining its idiosyncratic productivity z abroad.

Within a monopolistically competitive framework, a firm with idiosyncratic productivity z optimally chooses between three production modes, factoring in the effect of intangible investment on profits: domestic production with intangible investment, domestic production without intangible investment, and offshore production with intangible investment. Equations (7)–(9) present the profit functions for these modes.

$$\max_{\{\rho^I_{D,t}(z)\}} d^I_D(z) = \Big(\rho^I_{D,t}(z) - \psi^{\xi}_{D,t}\frac{w_t}{Z_t z}\Big)y_{D,t}(z) - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1) \tag{7}$$

$$\max_{\{\rho^{NI}_{D,t}(z)\}} d^{NI}_D(z) = \Big(\rho^{NI}_{D,t}(z) - \frac{w_t}{Z_t z}\Big)y_{D,t}(z) \tag{8}$$

$$\max_{\{\rho_{V,t}(z)\}} d_V(z) = \Big(\rho_{V,t}(z) - \tau Q_t\psi^{\xi}_{V,t}\frac{w^*_t}{Z^*_t z}\Big)y_{V,t}(z) - f_V Q_t\frac{w^*_t}{Z^*_t} - Q_t\phi_{V,t}\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t} - 1) \tag{9}$$

where $\rho^{NI}_{D,t}(z)$, $\rho^I_{D,t}(z)$, and $\rho_{V,t}(z)$ represent the relative prices for domestic production without intangible investment, domestic production with intangible investment, and offshore production with intangible investment, respectively. w_t and w^*_t denote real wages in the home and foreign markets, and Q_t represents the real exchange rate. Thus, the cost of producing a unit of output depends on whether production is domestic or offshore and whether intangible investment is undertaken. In other words, this decision is influenced not only by effective labor costs $\frac{w_t}{Z_t}$ and $\frac{w^*_t Q_t}{Z^*_t}$ across countries, but also by firm-specific labor productivity z and the marginal cost reduction fraction ψ_t.

Home firms investing in intangible capital incur fixed costs while gaining the ability to reduce marginal costs. For domestic producers, intangible investment lowers marginal costs by a factor of $\psi^{\xi}_{D,t}$; for offshore producers, by $\psi^{\xi}_{V,t}$. Following Gumpert, Manova, Rujan, and Schnitzer (2025), I assume $\psi_{D,t} > \psi_{V,t}$, reflecting higher fixed costs for intangible investment in an offshore plant. A lower $\psi_{V,t}$ indicates both higher fixed costs and greater marginal cost reduction efficiency for offshore firms. Fixed costs are independent of firm-specific productivity z. However, the fixed costs associated with intangible investment mean that only firms with relatively high idiosyncratic productivity z engage in such investment.

Moreover, home firms that offshore production incur a fixed cost of f_V units of foreign effective labor, covering the establishment and maintenance of the offshore facility. They also face an iceberg trade cost $\tau > 1$ when transporting goods produced overseas back to the home country.

The demand for the variety of firm z, produced domestically or offshore considering intangible investment, is $y^I_{D,t}(z) = \rho^I_{D,t}(z)^{-\theta}C_t,\ y^{NI}_{D,t}(z) = \rho^{NI}_{D,t}(z)^{-\theta}C_t$ or $y_{V,t}(z) = \rho_{V,t}(z)^{-\theta}\ C_t$ respectively, where C_t is aggregate Home consumption. Profit maximization yields equilibrium prices: $\rho^I_{D,t}(z) = \frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t z}$, $\rho^{NI}_{D,t}(z) = \frac{\theta}{\theta-1}\frac{w_t}{Z_t z}$ and $\rho_{V,t}(z) = \frac{\theta}{\theta-1}\tau Q_t\psi^{\xi}_{V,t}\ \frac{w^*_t}{Z^*_t z}$ for the three production methods. These prices show that firms with intangible capital investment price differently from non-intangible investing firms by a factor of ψ^{ξ}, allowing them to reduce prices proportionally to marginal cost reductions. The corresponding profits for each production cases are as follows: $d^I_{D,t}(z) = \frac{1}{\theta}\rho^I_{D,t}(z)^{1-\theta}C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1), d^{NI}_{D,t}(z) = \frac{1}{\theta}\rho^{NI}_{D,t}(z)^{1-\theta}C_t$ and $d_{V,t}(z) = \frac{1}{\theta}\rho_{V,t}(z)^{1-\theta}C_t - f_V Q_t\frac{w^*_t}{Z^*_t} - Q_t\phi_{V,t}\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t} - 1)$.

Each period, domestic firms decide whether to invest in intangible capital. A firm with productivity z compares domestic profit with intangible investment $d^I_{D,t}(z)$ and without such investment $d^{NI}_{D,t}(z)$. The cutoff productivity $z^I_{D,t}$ on the support interval $[z_{min}, \infty)$ is where the two profits are equal; at this threshold, the firm is indifferent between investing and not investing.

$$z^I_{D,t} = \{z : d^I_{D,t}(z^I_{D,t}) = d^{NI}_{D,t}(z^I_{D,t})\} \tag{10}$$

Similarly, a firm with productivity z determines its production location each period by comparing profits from domestic production with intangible investment, $d^I_{D,t}(z)$, and from offshoring, $d_{V,t}(z)$. The productivity cutoff $z_{V,t}$ on the support interval $[z_{min}, \infty)$ denotes the level at which the firm is indifferent, earning the same profit from either location.

$$z_{V,t} = \{z : d^I_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t})\} \tag{11}$$

The model implies that only relatively more productive domestic firms can invest in intangible capital, and even more productive firms among them can pursue offshore production with intangible investment. Since intangible investment entails fixed costs, only firms with productivity above the investment threshold ($z > z^I_{D,t}$) gain enough from intangible capital to offset these costs.[6]

Similarly, offshore production involves additional fixed and iceberg trade costs, such that only firms with idiosyncratic productivity above the offshoring cutoff ($z > z_{V,t}$) can generate sufficient gains to offset these costs. This implication is consistent with empirical evidences: multinational firms are more likely to invest in R&D (Castellani et al., 2015); and multinational firms are both more productive and earn higher R&D returns than domestic firms (Belderbos et al., 2014; Rahko, 2021). The Compustat Segment Historical data also backs up the finding that offshoring firms are more productive than non-offshoring firms, which is presented as stylized fact (iii) in Appendix A. Furthermore, the model's setup, where offshoring firms always undertake intangible investment and incur fixed costs, is motivated by stylized fact (iv) identified in Appendix A using Compustat Historical Segment data: offshoring firms are more likely to undertake intangible investments compared to domestic firms.

Additionally, the productivity cutoffs $z^I_{D,t}$ and $z_{V,t}$ respond to changes in relative effective

[6]Intangible investing firms are more productive, a fact supported by the Compustat Historical Segment data showing high average productivity for firms in the highest intangible quantile. See empirical fact (iii) and Figure 12 in Appendix A for details. Based on empirical fact (ii), intangible investing firms exhibit higher net sales.

labor costs across countries, thereby influencing the extensive margin over the business cycle. A decline in the relative cost of Home effective labor lowers domestic prices and raises Home firms' revenues, leading more Home firms to invest in intangibles at equilibrium. Similarly, a decline in the relative cost of Foreign effective labor increases the profitability of offshoring, resulting in a higher share of firms choosing to offshore. The existence and uniqueness of equilibrium productivity cutoffs are established in Appendix B.

2.3.4 Exporting

Firms in each economy can serve both domestic and foreign markets through exports, as in GM05. A Home firm with productivity z exports to the foreign market by using domestic labor $l_{X,t}$ to produce $y_{X,t}(z) = Z_t z l_{X,t}(z)$. Foreign firms similarly export to Home. Profit maximization yields the equilibrium export price and profit function for a Home exporter with productivity z: $\rho_{X,t}(z) = \frac{\theta}{\theta-1}\tau^* Q_t^{-1} \frac{w_t}{Z_t z}$ and $d_{X,t}(z) = \frac{1}{\theta}\rho_{X,t}(z)^{1-\theta} C_t^* Q_t - f_X \frac{w_t}{Z_t}$, where C_t^* is foreign aggregate consumption. Exporting generates additional profits but requires fixed costs (f_X) in units of Home effective labor and iceberg trade costs τ^*. Only Home firms with $z \geq z_{X,t}$ can export profitably, where $z_{X,t} = inf\{z \mid d_{X,t}(z) > 0\}$.

2.4 Aggregation over heterogeneous firms

This section reformulates the offshoring model with four representative Home firms: (1) domestic production without intangible investment, (2) domestic production with intangible investment, (3) offshore production with intangible investment, and (4) domestic production for export. Offshoring occurs only from Home to Foreign. Accordingly, Foreign has three representative firms: domestic production without intangible investment, domestic production with intangible investment, and exporters to Home.

2.4.1 Average productivity, prices, and profits

Figure 2 shows average productivity for three representative Home firms: those producing domestically without intangible investment, those producing domestically with intangible investment, and those producing offshore. Specifically, Figure 2 plots the density of firm-specific labor productivity z over the support interval $[z_{min}, \infty)$. $N_{D,t}^{NI}$ firms with productivity $z < z_D^{NI}$ produce domestically without intangible investment, and their average productivity is $\tilde{z}_{D,t}^{NI}$. Additionally, $N_{D,t}^{I}$ firms with productivity $z_{D,t}^{I} < z < z_{V,t}$ have an average productivity of $\tilde{z}_{D,t}^{I}$. Finally, $N_{V,t}$ firms with productivity ($z > z_{V,t}$) have an average productivity of $\tilde{z}_{V,t}$. Given that firm-specific labor productivity levels z are randomly drawn from a common distribution $G(z)$ with density $g(z)$, the three average productivity levels are:

$$\tilde{z}_{D,t}^{NI} = \left[\frac{1}{G(z_{D,t}^{I})}\int_{z_{min}}^{z_{D,t}^{I}} z^{\theta-1} g(z)dz\right]^{\frac{1}{\theta-1}}, \quad \tilde{z}_{D,t}^{I} = \left[\frac{1}{G(z_{V,t}) - G(z_{D,t}^{I})}\int_{z_{D,t}^{I}}^{z_{V,t}} z^{\theta-1} g(z)dz\right]^{\frac{1}{\theta-1}},$$

$$\tilde{z}_{V,t} = \left[\frac{1}{1 - G(z_{V,t})} \int_{z_{V,t}}^{\infty} z^{\theta-1} g(z) dz \right]^{\frac{1}{\theta-1}} \tag{12}$$

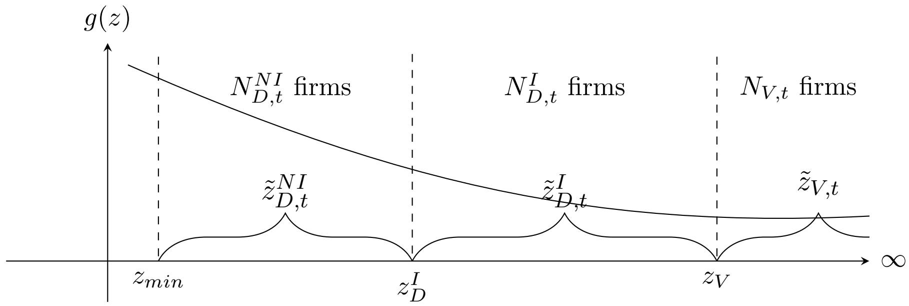

Figure 2: Average productivity of firms producing at Home

Assuming firm-specific labor productivity z follows a Pareto distribution with density $g(z) = kz^k_{min}/z^{k+1}$ and cumulative distribution $G(z) = 1 - (z_{min}/z)^k$ over $[z_{min}, \infty)$, the average productivity levels can be expressed in terms of the cutoffs $z^I_{D,t}$ and $z_{V,t}$:

$$\tilde{z}^{NI}_{D,t} = \nu z_{min} z^I_{D,t} \left[\frac{z^{I\ k-(\theta-1)}_{D,t} - z^{k-(\theta-1)}_{min}}{z^{I\ k}_{D,t} - z^k_{min}} \right]^{\frac{1}{\theta-1}}, \quad \tilde{z}^I_{D,t} = \nu z^I_{D,t} z_{V,t} \left[\frac{z^{k-(\theta-1)}_{V,t} - z^{I\ k-(\theta-1)}_{D,t}}{z^k_{V,t} - z^{I\ k}_{D,t}} \right]^{\frac{1}{\theta-1}},$$

$$\tilde{z}_{V,t} = \nu z_{V,t} \tag{13}$$

where the cutoff is $z_{V,t} = z_{min}(N_t/N_{V,t})^{1/k}$, with parameters $\nu \equiv \left[\frac{k}{k-(\theta-1)}\right]^{\frac{1}{\theta-1}}$ and $k > \theta - 1$.

Foreign firms exclusively serve their domestic market through domestic production, with average productivity given by:

$$\tilde{z}^{NI*}_{D,t} = \nu z^*_{min} z^{I*}_{D,t} \left[\frac{z^{I*\ k-(\theta-1)}_{D,t} - z^{*k-(\theta-1)}_{min}}{z^{I*\ k}_{D,t} - z^{*k}_{min}} \right]^{\frac{1}{\theta-1}}, \quad \tilde{z}^{I*}_{D,t} = \nu z^{I*}_{D,t} \tag{14}$$

The average productivity levels of exporting firms are similar to GM05:

$$\tilde{z}_{X,t} = \nu z_{min} \left(\frac{N_t}{N_{X,t}} \right)^{1/k} \text{ and } \tilde{z}^*_{X,t} = \nu z^*_{min} \left(\frac{N^*_{D,t}}{N^*_{X,t}} \right)^{1/k} \tag{15}$$

Based on the average productivity of firms engaged in domestic production (with and without intangible investment), offshoring, and exporting, Table 1 presents the average prices and profits for each production type. Therefore, the aggregate price indexes for Home and Foreign are: $1 = N^{NI}_{D,t}(\tilde{\rho}^{NI}_{D,t}(z))^{1-\theta} + N^I_{D,t}(\tilde{\rho}^I_{D,t}(z))^{1-\theta} + N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta} + N^*_{X,t}(\tilde{\rho}^*_{X,t}(z^*))^{1-\theta}$ and $1 = N^{*NI}_{D,t}(\tilde{\rho}^{*NI}_{D,t}(z^*))^{1-\theta} + N^{*I}_{D,t}(\tilde{\rho}^{*I}_{D,t}(z^*))^{1-\theta} + N_{X,t}(\tilde{\rho}_{X,t}(z))^{1-\theta}$. Similarly, the total profits of firms from Home and Foreign are $N_t \tilde{d}_t(z) = N^{NI}_{D,t} \tilde{d}^{NI}_{D,t}(z) + N^I_{D,t} \tilde{d}^I_{D,t}(z) + N_{V,t} \tilde{d}_{V,t}(z) + N_{X,t} \tilde{d}_{X,t}(z)$ and $N^*_t \tilde{d}^*_t(z^*) = N^{*NI}_{D,t} \tilde{d}^{*NI}_{D,t}(z^*) + N^{*I}_{D,t} \tilde{d}^{*I}_{D,t}(z^*) + N^*_{X,t} \tilde{d}^*_{X,t}(z^*)$.

Origin	Production	Market	Intangible	Average Prices	Average Profits
Home	Home	Home	Y	$\tilde{\rho}^I_{D,t} = \frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t\tilde{z}^I_{D,t}}$	$\tilde{d}^I_{D,t} = \frac{1}{\theta}{\tilde{\rho}^I_{D,t}}^{1-\theta}C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1)$
Home	Home	Home	N	$\tilde{\rho}^{NI}_{D,t} = \frac{\theta}{\theta-1}\frac{w_t}{Z_t\tilde{z}^{NI}_{D,t}}$	$\tilde{d}^{NI}_{D,t} = \frac{1}{\theta}{\tilde{\rho}^{NI}_{D,t}}^{1-\theta}C_t$
Home	Foreign	Home	Y	$\tilde{\rho}_{V,t} = \frac{\theta}{\theta-1}\tau Q_t\psi^{\xi}_{V,t}\frac{w^*_t}{Z^*_t\tilde{z}_{V,t}}$	$\tilde{d}_{V,t} = \frac{1}{\theta}\tilde{\rho}^{1-\theta}_{V,t}C_t - f_V Q_t\frac{w^*_t}{Z^*_t} - Q_t\phi_{V,t}\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t} - 1)$
Home	Home	Foreign	N	$\tilde{\rho}_{X,t} = \frac{\theta}{\theta-1}\tau^* Q^{-1}_t\frac{w_t}{Z_t\tilde{z}_{X,t}}$	$\tilde{d}_{X,t} = \frac{1}{\theta}\tilde{\rho}^{1-\theta}_{X,t}C^*_t Q_t - f_X\frac{w_t}{Z_t}$
Foreign	Foreign	Foreign	Y	$\tilde{\rho}^{*I}_{D,t} = \frac{\theta}{\theta-1}\psi^{*\xi*}_{D,t}\frac{w^*_t}{Z^*_t\tilde{z}^{*I}_{D,t}}$	$\tilde{d}^{*I}_{D,t} = \frac{1}{\theta}{\tilde{\rho}^{*I}_{D,t}}^{1-\theta}C^*_t - \phi^*_{D,t}\frac{w^*_t}{Z^*_t}({\psi^*_{D,t}}^{-\xi} - 1)$
Foreign	Foreign	Foreign	N	$\tilde{\rho}^{*NI}_{D,t} = \frac{\theta}{\theta-1}\frac{w^*_t}{Z^*_t\tilde{z}^{*NI}_{D,t}}$	$\tilde{d}^{*NI}_{D,t} = \frac{1}{\theta}{\tilde{\rho}^{*NI}_{D,t}}^{1-\theta}C^*_t$
Foreign	Foreign	Home	N	$\tilde{\rho}^*_{X,t} = \frac{\theta}{\theta-1}\tau Q_t\frac{w^*_t}{Z^*_t\tilde{z}^*_{X,t}}$	$\tilde{d}^*_{X,t} = \frac{1}{\theta}{\tilde{\rho}^*_{X,t}}^{1-\theta}C_t Q^{-1}_t - f^*_X\frac{w^*_t}{Z^*_t}$

Table 1: Average prices and profits

2.4.2 Indifference conditions for offshoring and exporting

At the productivity cutoff, the two production strategies yield equivalent average productivity levels, establishing a linkage between the average profits of firms at distinct productivity cutoffs. First, at the intangible investment cutoff $z^I_{D,t}$, a connection emerges between the average profits of domestic firms with and without intangible investment. Second, at the offshoring cutoff $z_{V,t}$, a link appears between the average productivity of offshore firms and those producing domestically with intangible investment.

$$
\begin{aligned}
\tilde{d}^{\,I}_{D,t} =& \frac{k}{k-(\theta-1)}\left(\frac{z^I_{D,t}z_{V,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1}\left[\frac{z^{k-\theta+1}_{V,t} - {z^I_{D,t}}^{k-\theta+1}}{z^k_{V,t} - {z^I_{D,t}}^k}\right]\tilde{d}^{NI}_{D,t} \\
&+ \left\{\frac{k}{k-(\theta-1)}z^{\theta-1}_{V,t}\left[\frac{z^{k-\theta+1}_{V,t} - {z^I_{D,t}}^{k-\theta+1}}{z^k_{V,t} - {z^I_{D,t}}^k}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1)
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
\tilde{d}_{V,t} =& \frac{k}{k-(\theta-1)}\left(\frac{z_{V,t}}{\tilde{z}^I_{D,t}}\right)^{\theta-1}\left\{\tilde{d}^I_{D,t} + \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1)\right\} - \frac{k}{k-(\theta-1)}\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1) \\
&+ \frac{\theta-1}{k-(\theta-1)}\left\{f_V Q_t\frac{w^*_t}{Z^*_t} + \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_V - 1)\right\}
\end{aligned}
\tag{17}
$$

Additionally, since firms at the productivity cutoff $z_{X,t}$ earn zero profits from exporting, the average export profits are derived as shown in GM05:

$$
\tilde{d}_{X,t} = \frac{\theta-1}{k-(\theta-1)}f_X\frac{w_t}{Z_t}, \quad \tilde{d}^*_{X,t} = \frac{\theta-1}{k-(\theta-1)}f^*_X\frac{w^*_t}{Z^*_t} \tag{18}
$$

The derivations of equations (16)-(18) are provided in Appendix E.

2.5 Household Budget Constraint and Intertemporal Choices

The representative Home household entering period t holds two types of assets: domestic risk-free bond holdings B_t, and mutual fund share holdings x_t. Each period, the household begins with share holdings x_t in a mutual fund of N_t firms with average market value $\tilde{v}_t$. It also holds Home risk-free real bonds, B_t, denominated in units of the Home consumption basket. The household receives dividends equal to the average firm profit $\tilde{d}_t$ in proportion with the stock of firms N_t, the real wage w_t for $L \equiv 1$ supplied inelastically, and real rates of return r_t from the Home bonds. The household allocates these resources between purchases of bonds and shares to be carried into next period and consumption. Each period, the household purchases two types of assets, risk-free bonds B_{t+1} and mutual fund shares x_{t+1}. The mutual fund comprises N_t incumbent Home firms (producing domestically or offshore) and $N_{E,t}$ new entrants in period t. The average market value of each firm is $\tilde{v}_t$, representing the net present value of expected future profits. When the government provides subsidies or collect tariffs, the revenues are rebated to households in a lump-sum.

The period budget constraint (in units of consumption) is:

$$(1 + r_t)B_t + (\tilde{v}_t + \tilde{d}_t)N_t x_t + w_t L + T_t \geq B_{t+1} + \tilde{v}_t(N_t + N_{E,t})x_{t+1} + C_t \tag{19}$$

The household maximizes its expected intertemporal utility subject to equation (19). The Euler equations for bonds is:

$$C_t^{-\gamma} = \beta(1 + r_{t+1})E_t(C_{t+1})^{-\gamma} \tag{20}$$

The Euler equations for stocks is as follows, with the rate of firm exit δ described in Section 2.2:

$$\tilde{v}_t = \beta(1 - \delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\tilde{v}_{t+1} + \tilde{d}_{t+1})\right] \tag{21}$$

With the consumption basket C_t set as the numeraire, the Home price index is normalized such that $1 = \left(\int_{\Omega_t} \rho_t(\omega)^{1-\theta} d\omega\right)^{\frac{1}{1-\theta}}$.

2.6 Aggregation over heterogeneous firms

In a financial autarky, household income from labor, stocks, and bonds is used for consumption and investment in new firms within each country:

$$C_t + N_{E,t}\tilde{v}_t + B_{t+1} = w_t L + N_t\tilde{d}_t + (1 + r_t)B_t \tag{22}$$

$$C_t^* + N_{E,t}^*\tilde{v}_t^* + B_{t+1}^* = w_t^* L^* + N_{D,t}^*\tilde{d}_t^* + (1 + r_t^*)B_t^* \tag{23}$$

The current account in the Home is given by:

$$CA_t = \underbrace{N_{X,t}(\tilde{\rho}_{X,t}(z))^{1-\theta}C_t^*Q_t}_{\text{(a) Exports}} + \underbrace{N_{V,t}\tilde{d}_{V,t}(z)}_{\text{(b) Repatriated offshore profits}} - \underbrace{N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta}C_t^*Q_t}_{\text{(c) Value added offshore in Foreign}} - \underbrace{N_{X,t}^*(\tilde{\rho}_{X,t}^*(z^*))^{1-\theta}C_t}_{\text{(d) Imports from Foreign}} \tag{24}$$

Under financial autarky, the balanced current account condition ($CA_t = 0$) requires that the sum of (a) export value added and (b) repatriated offshore profits equals the sum of (c) the value added by offshoring affiliates imported into Home and (d) imports from Foreign firms.

As a result, the benchmark model of financial autarky for the Home economy consists of 19 equations and 19 endogenous variables: $N_t, N_{D,t}^I, N_{D,t}^{NI}, N_{V,t}, N_{X,t}, N_{E,t}, \tilde{d}_t, \tilde{d}_{D,t}^I, \tilde{d}_{D,t}^{NI}, \tilde{d}_{V,t}, \tilde{d}_{X,t}, \tilde{z}_{D,t}^I, \tilde{z}_{D,t}^{NI}, \tilde{z}_{V,t}, \tilde{z}_{X,t}, \tilde{v}_t, r_t, w_t, C_t$.

Since Foreign firms do not produce in Home, the Foreign economy reduces to 16 equations and variables, with no counterparts for $N_{V,t}, \tilde{d}_{V,t}, \tilde{z}_{V,t}$. The domestic productivity of a representative Foreign firm $\tilde{z}_{D,t}^*$ is constant over time.

3 Reshoring Policies

This paper analyzes three reshoring policies: (1) Lump-sum intangible subsidies, (2) Tax credits, and (3) Tariffs. Using New Industrial Policy Observatory (NIPO) data covering over 2,500 industrial policies announced globally in 2023, Evenett, Jakubik, Martín, and Ruta (2024) find that subsidies are the most dominant policy instrument, followed by trade restrictions. Nearly half of the industrial policies in advanced economies are subsidies—most commonly direct R&D grants and tax credits—that frequently target R&D and high-tech (intangible-intensive) industries. In this paper, direct R&D grants are defined as the lump-sum intangible subsidies in Section 3.1.1, and tax credits are defined as the tax credit in Section 3.1.2. While lump-sum subsidies are output-independent grants, tax credits are provided proportionally to production.

A representative example of lump-sum subsidies is the $7.86 billion direct grant awarded to Intel under the CHIPS Act for its Oregon R&D hub and Megafab construction in Arizona and Ohio. Intel is actively pursuing R&D reshoring through a strategic reallocation of capital—pausing its major expansion in Israel (Fab 38) and implementing layoffs while concentrating resources on the D1X R&D facility in Oregon. Intel also announced the suspension of its foundry projects in Germany and Poland, while prioritizing the uninterrupted progress of its US Megafab projects in Arizona and Ohio (Intel CEO Pat Gelsinger's Message to Employees, 2024.09.16).

A representative example of tax-based incentives is the immediate expensing of Research and Experimental (R&E) expenditures under Section 174A of the One Big Beautiful Bill Act

(2025). This policy serves as a powerful reshoring incentive by allowing for the immediate deduction of tax-related variable costs when R&D personnel and facilities are relocated to the United States. While R&D conducted abroad requires 15 years for capitalization and amortization, R&D performed within U.S. territory is eligible for immediate expensing.

In this paper, tax credits are modeled as reducing a firm's marginal cost; although they do not directly subsidize marginal costs in practice, they are applied to variable production costs, thereby lowering the firm's real tax burden and effective marginal cost. This formulation accurately reflects output-proportional reshoring subsidies—such as the IRA Section 45X Advanced Manufacturing Production Credit—and effectively captures the mechanism that lowers prices and increases profits for reshoring firms.

Meanwhile, the most frequently utilized trade restrictions are import and export barriers, with tariffs emerging as the most representative import barrier within industrial policy, which is introduced in Section 3.1.3 of this paper. To illustrate how tariffs function as reshoring policies, the Trump administration recently began imposing strategic tariffs under Section 301 on intangible-intensive goods whose Rules of Origin indicate foreign production. As a consequence of this policy, the battery tariffs levied under Section 301 deteriorated the profitability of Lithium-Iron Phosphate (LFP) battery production at Tesla's Shanghai factory. In response, Tesla confirmed the establishment of a new LFP battery plant in Nevada, clearly demonstrating that tariffs effectively operate as a reshoring policy.

3.1 Reshoring Policies

3.1.1 Lump-sum Intangible Subsidies

Lump-sum intangible investment subsidies are implemented by reducing the fixed costs for reshoring firms when they expand their domestic intangible investment. This increases the profit of domestic intangible firms by the subsidy amount ($\eta > 0$), modifying the profit function as shown in equation (25). These subsidies do not directly alter revenue or cost variables linked to heterogeneous productivity; they solely incentivize domestic intangible investment. The benefit applies exclusively to firms transitioning from offshoring back to domestic production ($z \in [z^I_{V,t}, z^R_{V,t}]$).

$$d^R_{D,t}(z) = \frac{1}{\theta}\left(\rho^R_{D,t}(z)\right)^{1-\theta} C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1) + \eta_t \qquad \text{where } z \in [z^I_{V,t}, z^R_{V,t}] \tag{25}$$

The impact of these subsidies is illustrated in Figure 3. The subsidy shifts the profit function of domestic intangible firms vertically upward ($d_{D,t}(z)^R$). The subsidy shifts the profit function for domestic firms investing in intangibles vertically upward ($d_{D,t}(z)^R$), inducing firms between the pre-policy offshore cutoff $z^I_{V,t}$ and the post-policy cutoff $z^R_{V,t}$ to reshore. Firms with productivity exceeding the new threshold ($z^R_{V,t}$) continue offshoring regardless of

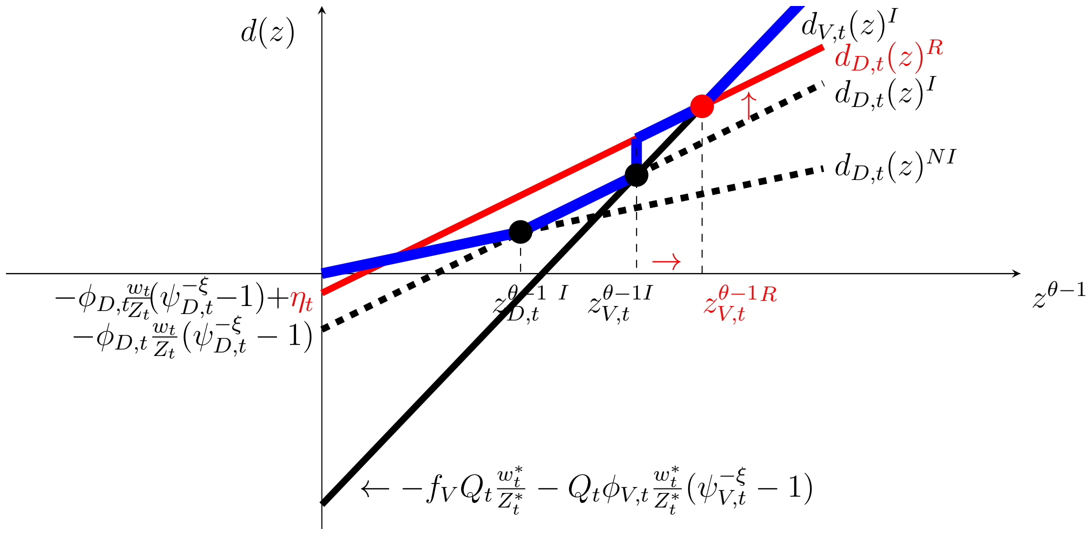

Figure 3: Effects of Lump-sum Intangible Subsidies on Firm Profits

the policy. Firms with productivity below the original threshold ($z_{V,t}^I$) maintain domestic production but do not qualify for the subsidy. Specifically, in productivity ranges below the domestic intangible investment threshold ($z_{D,t}^I$), domestic production without intangibles provides the highest profit. Within the interval $[z_{D,t}^I, z_{V,t}^I]$, domestic production with intangible investment generates the greatest profit. Firms in the $[z_{V,t}^I, z_{V,t}^R]$ range—which previously engaged in offshoring but transitioned to reshoring—attain maximum profit through domestic production with intangible investment. Firms with productivity above the post-policy offshore cutoff continue to maximize profit through offshoring.

3.1.2 Tax Credits

Tax credits are implemented for reshoring intangible-intensive firms by lowering variable costs as production increases, thereby enhancing profitability. Rather than directly subsidizing marginal costs, the tax credit represents a credit against variable costs incurred during the production process. Similar to the IRA Section 45X Advanced Manufacturing Production Credit, this reduces a firm's real tax burden in proportion to its output, effectively lowering its effective marginal cost. The tax credit increases the profits of domestic intangible-intensive firms more significantly as production expands ($t_s > 0$), modifying their profit function as expressed in equation (26).

Reshoring firms incorporate the government's tax credit into their profit maximization process to determine the optimal price, which subsequently decreases by the subsidy rate. Substituting this optimal price back into the profit expression (26) yields equation (27); this demonstrates that the slope of the profit function becomes steeper compared to that of existing domestic intangible firms. In other words, the reshoring tax credit allows higher-productivity firms to realize greater profit improvements, providing them with a stronger incentive to expand production. The benefits of these subsidies apply exclusively to firms that have transitioned from offshoring back to domestic production ($z \in [z^I_{V,t}, z^R_{V,t}]$).

$$\max_{\{\rho_{D,t}\}} d^R_{D,t}(z) = \left(\rho_{D,t} - \frac{1}{1+t_{s,t}}\psi^{\xi}_{D,t}\frac{w_t}{Z_t z}\right)\rho^{-\theta}_{D,t}C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1) \tag{26}$$

Profit maximization: $\rho^R_{D,t} = \frac{1}{1+t_{s,t}}\frac{\theta}{\theta-1}\psi^{\xi}_{D}\frac{w_t}{Z_t z}$ where $z \in [z^I_{V,t}, z^R_{V,t_{t_s}}]$

Rewrite profit function: $$d^R_{D,t}(z) = \frac{1}{\theta}(1+t_{s,t})^{\theta-1}\left(\frac{\theta}{\theta-1}\psi^{\xi}_{D}\frac{w_t}{Z_t z}\right)^{1-\theta}C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1) \tag{27}$$

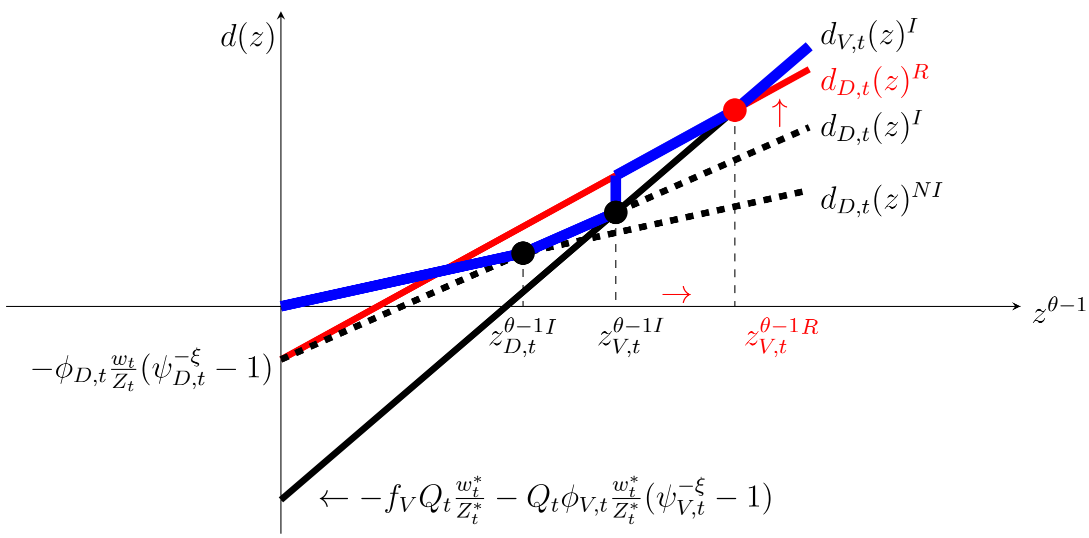

Figure 4: Effects of Tax Credit on Firm Profits

The effects of the tax credit are illustrated in Figure 4. The tax credit steepens the slope of the profit function for domestic intangible firms ($d_{D,t}(z)^R$), inducing firms between the pre-policy offshore cutoff $z^I_{V,t}$ and the post-policy cutoff $z^R_{V,t}$ to reshore. Firms with productivity above $z^R_{V,t}$ continue offshoring, while those below $z^I_{V,t}$ maintain domestic production with intangibles but remain ineligible for subsidies. Similar to the case of lump-sum subsidies, domestic production without intangible investment yields the highest profit below $z^I_{D,t}$, and domestic production with intangible investment is most profitable within $[z^I_{D,t}, z^I_{V,t}]$. Firms above $z^R_{V,t}$ maximize profits via offshore production, whereas those in the $[z^I_{V,t}, z^R_{V,t}]$ range—previously offshoring—now maximize profits through domestic production with intangible investment.

3.1.3 Tariffs

Tariffs serve as reshoring policies that induce a return to domestic production by lowering the profits of offshoring firms and reducing the domestic market share of foreign exporters through an increase in their iceberg trade costs. Specifically, tariffs raise the prices of offshore firms and import prices by the tariff rate $(1+\tau^R)$, thereby diminishing the gains from foreign production. As a result, firms that previously offshored initiate domestic production, and home consumers substitute imported varieties with domestically produced ones. Tariffs induce reshoring by disincentivizing foreign production, a mechanism distinct from subsidies that encourage domestic production by intangible-intensive firms. This tariff increase applies to firms that previously engaged in offshoring but have since returned to domestic production $(z \in [z^I_{V,t}, z^R_{V,t}])$.

$$\max_{\{\rho_{V,t}\}} d^R_{V,t}(z) = \left(\rho_{V,t}(z) - (1+\tau^R_t)\tau Q_t \psi^{\xi}_{V,t} \frac{w^*_t}{Z^*_t z}\right) y_{V,t}(z) - f_V Q_t \frac{w^*_t}{Z^*_t} - Q_t \phi_{V,t} \frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t} - 1) \tag{28}$$

$$\text{Profit maximization for offshore firms: } \rho^R_{V,t} = \frac{\theta}{\theta-1}(1+\tau^R_t)\tau Q_t \psi^{\xi}_{V,t} \frac{w^*_t}{Z^*_t z} \tag{29}$$

$$\max_{\{\rho^*_{X,t}\}} d^{*R}_{X,t}(z) = \left(\rho^*_{X,t}(z) - (1+\tau^R_t)\tau Q_t \frac{w^*_t}{Z^*_t z^*}\right) y^*_{X,t}(z) Q^{-1}_t - f^*_X \frac{w^*_t}{Z^*_t} \tag{30}$$

$$\text{Profit maximization for Foreign export firms: } \rho^{*R}_{X,t} = \frac{\theta}{\theta-1}(1+\tau^R_t)\tau Q_t \frac{w^*_t}{Z^*_t z^*} \tag{31}$$

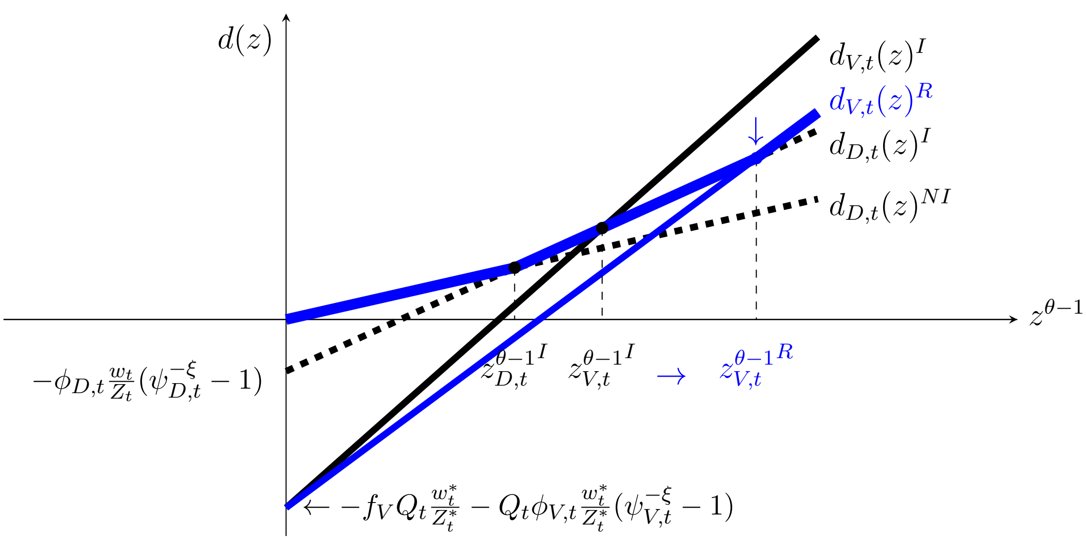

Figure 5: Effects of Tariffs on Firm Profits

The impact of tariffs is shown in Figure 5. The imposition of tariffs flattens the slope of the offshoring profit function $(d_{V,t}(z)^R)$; thus, firms between the pre-policy offshore cutoff $z^I_{V,t}$ and the post-policy cutoff $z^R_{V,t}$ choose to reshore to avoid tariff costs. Firms with productivity above $z^R_{V,t}$ maintain offshoring, while those below $z^I_{V,t}$ continue domestic production.

Consistent with the subsidy scenarios, domestic production without intangible investment provides the highest profit for productivity levels below $z^I_{D,t}$, and domestic production with intangible investment yields the highest profit within $[z^I_{D,t}, z^I_{V,t}]$. Firms above $z^R_{V,t}$ maximize profits through offshore production. Firms in the $[z^I_{V,t}, z^R_{V,t}]$ range—formerly offshorers—now maximize profits via domestic production with intangible investment due to the tariffs. Notably, unlike the two subsidies, the profits of reshoring firms decrease compared to their previous offshoring state.

3.1.4 Government

All net revenues or expenditures from the three reshoring policies are rebated to households as a lump-sum transfer T_t, as specified in Equation (19). The expenditures and revenues for each policy are defined as follows. When the government provides lump-sum intangible subsidies to reshoring firms, the total expenditure is the product of the subsidy amount, η_t, and the number of reshoring firms receiving the grant, $N^R_{D,t}$ ($T_t = -\eta_t N^R_{D,t}$). In the case of tax credit subsidies, the total government expenditure T_t is calculated by multiplying the per-unit subsidy, $\frac{t_s}{1+t_s}$, by the number of reshoring firms, $N^R_{D,t}$, and their average revenue, $\tilde{\rho}^R_{D,t}\tilde{y}^R_{D,t}$ ($T_t = -\frac{t_s}{1+t_s} N^R_{D,t}\tilde{\rho}^R_{D,t}\tilde{y}^R_{D,t} = -\frac{t_s}{1+t_s} N^R_{D,t}\tilde{\rho}^R_{D,t}\left(\frac{\tilde{\rho}^R_{D,t}}{1+t_s}\right)^{-\theta} C_t$). The demand function $\tilde{y}^R_{D,t}$ incorporates a lower price because the tax credit subsidy reduces the final amount paid by consumers. The per-unit subsidy is derived from the wedge between the consumer price and the producer price.

Unlike the two subsidy types that represent fiscal expenditures, tariff collection constitutes government revenue. When the government imposes tariffs τ^R on offshoring firms and foreign exporting firms, it generates revenue equal to the product of the tariff rate, the number of firms, and their average revenue. Specifically, revenue collected from offshoring firms is $\tau^R N_{V,t}\tilde{\rho}^R_{V,t}\tilde{y}_{V,t}$, and revenue from foreign exporting firms is $\tau^R N^*_{X,t}\tilde{\rho}^{*R}_{X,t}\tilde{y}^*_{X,t}$. Therefore, the final government revenue from tariffs is: $T_t = \tau^R N_{V,t}\tilde{\rho}^R_{V,t}\tilde{y}_{V,t} + \tau^R N^*_{X,t}\tilde{\rho}^{*R}_{X,t}\tilde{y}^*_{X,t} = \tau^R N_{V,t}\tilde{\rho}^R_{V,t}[(1+\tau^R)\tilde{\rho}_{V,t}]^{-\theta}C_t$ $+\tau^R N^*_{X,t}\tilde{\rho}^{*R}_{X,t}[(1+\tau^R)\tilde{\rho}^*_{X,t}]^{-\theta}C_t$. The higher price in the demand functions $\tilde{y}_{V,t}$ and $\tilde{y}^*_{X,t}$ reflects the increased costs borne by consumers due to the tariff imposition.

4 Model Calibration

4.1 Parameterization

The model calibrates $\beta = 0.99$ to match a 4% annual interest rate, and the relative risk aversion coefficient is $\gamma = 2$. Consistent with GM05, the intra-temporal elasticity of substitution (θ), the firm exit probability (δ), and the iceberg trade cost (τ) are set to 3.8, 0.025, and 1.2, respectively. Additionally, the Pareto distribution parameter (k) is calibrated to 3.8.

Following Zlate16, the fixed costs of offshoring (f_V) and exporting (f_X and f^*_X) are set to 0.095, 0.040, and 0.025, respectively. These parameter values ensure that the steady-state model replicates the observed significance of offshoring in the Foreign economy, as well as the role of exports in both the Foreign and Home economies. The lower bound of productivity support is normalized to $z_{min} = z^*_{min} = 1$.

According to Zlate16, the sunk entry cost is set to be higher in Foreign than in Home ($f^*_E = 4f_E$ and $f_E = 1$) to reflect the greater regulatory burden of starting a business abroad, thereby introducing an asymmetry in the effective labor costs between the two countries in steady state. Consequently, the steady-state output, the number of firms, the labor demand, and the effective wage are relatively lower in Foreign. This calibration is consistent with the 1.5 times higher cost of business start-up procedures in China compared to the United States in purchasing power parity terms in 2015 (World Bank Database, 2015).

The calibrated values for k, τ, f_V, f_X, f^*_X, along with asymmetric sunk entry costs, result in a steady-state value for the terms of labor that is less than one ($TOL = \frac{Qw^*/Z^*}{w/Z} = 0.76$). This indicates that the steady-state cost of effective labor in Foreign is 76% of that in Home, providing an incentive for some Home firms to offshore production.

Following De Ridder24's estimates of intangible cost share (ϕ), I set $\phi = 0.215$ (Home) and $\phi^* = 0.279$ (Foreign), reflecting higher US intangible investment efficiency. I assume US R&D efficiency (ϕ) is lower than trading partners (ϕ^*), using France's (ϕ^*) as a proxy for Foreign. The intangible cost elasticities, ξ and ξ^*, are set to 2, following De Ridder24. Setting these to 0 nests the model in Zlate16's framework without intangible investment.

Parameter	Description	Value	Source
β	Discount rate	0.99	GM05
γ	The coefficient of relative risk aversion	2	GM05
θ	The intra-temporal elasticity of substitution	3.8	GM05
δ	The probability of firm exit	0.025	GM05
k	The Pareto distribution parameter	3.8	
τ	The iceberg trade cost	1.2	Zlate16
f_E	The sunk entry cost, Home	1	Zlate16
f^*_E	The sunk entry cost, Foreign	4	Zlate16
f_V	The fixed costs of offshoring	0.095	Zlate16
f_X	The exporting fixed cost, Home	0.040	Zlate16
f^*_X	The exporting fixed cost, Foreign	0.025	Zlate16
ξ, ξ^*	Intangibles cost elasticity	2	De Ridder24
ϕ	Share of intangible costs in fixed costs, Home	0.215	De Ridder24
ϕ^*	Share of intangible costs in fixed costs, Foreign	0.279	De Ridder24
ψ_D	The reduction fraction of marginal costs, Home	0.96	Compustat Segment
ψ_V	The reduction fraction of marginal costs for offshoring, Home	0.93	Compustat Segment
ψ^*_D	The reduction fraction of marginal costs, Foreign	0.96	Compustat Segment

Table 2: Calibration parameters and steady-state targets

Follwing Lee (2025), the reduction fraction of marginal costs for Home and Foreign firms with intangible investment is set at 0.96. This indicates that these firms can reduce their marginal costs by $1 - \psi^{\xi}_{D,t} = 1 - 0.96^2 = 0.078$, or 7.8%. Estimated using Compustat Segment data, the event study indicates a 7% decline in marginal cost at the timing of the event when

U.S. manufacturing firms increased their intangible investments (see Appendix A, Table 4, column (2) for details). For simplicity of comparison, I assume that foreign firms experience the same rate of marginal cost reduction from intangible investments as domestic firms.

The reduction fraction for Home firms engaged in offshoring is set at 0.93, implying a marginal cost reduction of $1 - \psi_{V,t}^{\xi} = 1 - 0.93^2 = 0.135$, or 13.5%. Following Gumpert et al. (2025), this paper assumes $\psi_{D,t} > \psi_{V,t}$; therefore offshore intangible investment requires higher fixed costs than domestic intangible investment. Estimated using Compustat Segment data, the event study indicates a 13.6% decline in marginal cost at the timing of the event when U.S. offshoring manufacturing firms increased their intangible investments (see Appendix A, Table 4, column (6) for details).

Therefore, the most productive offshoring firms can achieve an average marginal cost reduction of 13.5% through intangible investment, while domestic Home firms can reduce costs by 7.8%, and Foreign domestic firms by 7.8%. All parameterizations are summarized in Table 2.

4.2 Calibration Target

To evaluate the macroeconomic effects of the three reshoring policies described above, this paper calibrates the policy parameters to ensure that the total tax expenditure is equal across all experiments. Specifically, the targets are set such that the cumulative absolute sum of the tax expenditure (revenue), as defined in Section 3.1.4, is equal across all policies over a 10-year period of permanent shocks.

5 Results

5.1 Impulse responses

To illustrate the aggregate effects of reshoring policies, I log-linearize the model around the steady state and compute impulse responses to a permanent 1% reshoring policy shock in the Home economy. The CHIPS Act's Section 48D investment tax credit includes a ten-year guardrail restricting beneficiary firms from expanding production capacity in China. Motivated by this statutory horizon, we model the policy as a persistent distortion to firms' offshoring decisions that remains effective for ten years.

In this paper, reshoring policies are designed to expand the share of firms engaged in domestic production with intangible investment. Specifically, the two types of subsidies—lump-sum and tax credits—incentivize domestic intangible investment, while tariffs are structured to disincentivize foreign production. By inducing an increase in domestic production, all reshoring policies stimulate Home labor demand and push up Home wages, thereby leading to an appreciation of the Terms of Labor (TOL). The TOL, defined as $TOL = \frac{Q_t w_t^*/Z_t^*}{w_t/Z_t}$, represents the relative cost of effective labor across countries; a rise in Home wages relative to Foreign wages results in an appreciation, characterized by a decrease in the TOL value, as illustrated in Figure 6. When calibrating each policy to match the same cumulative increase in tax over 100 periods, the results indicate that the TOL appreciates consistently across all policy frameworks.

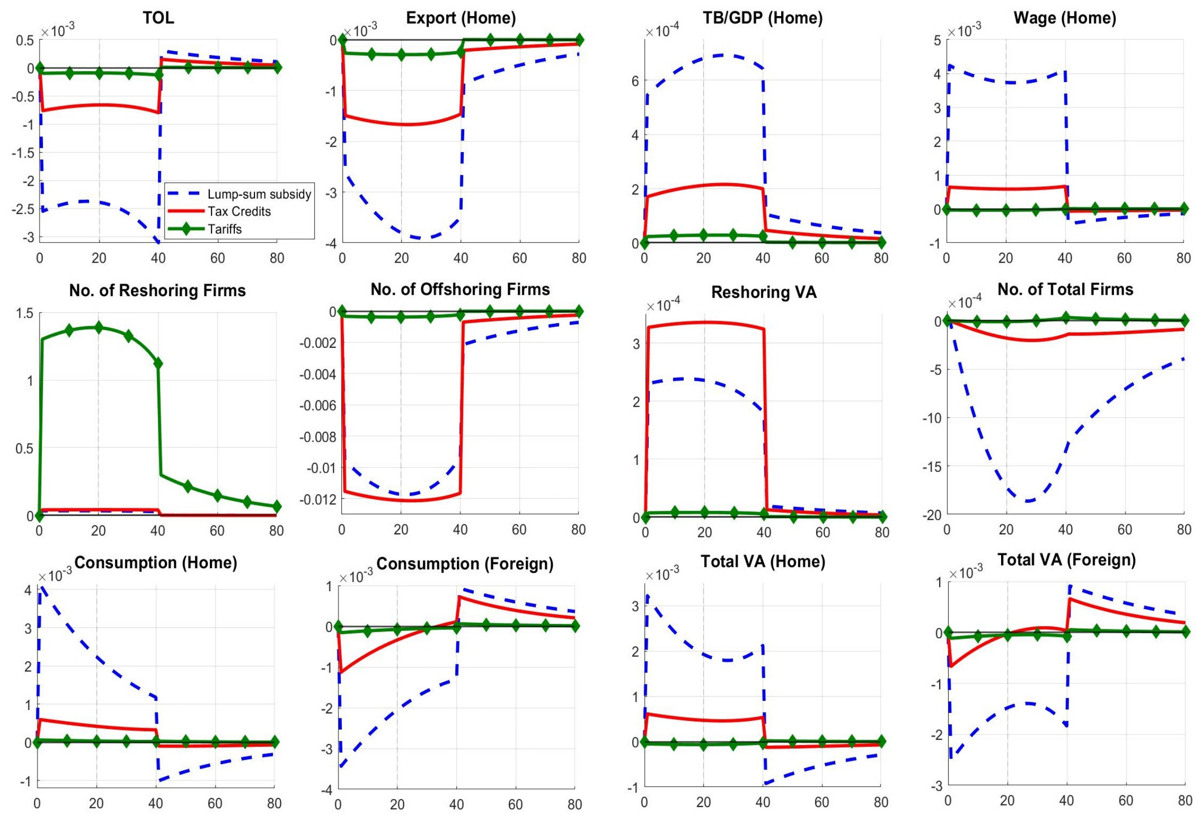

Figure 6: Macroeconomic Impacts of Reshoring Policies (Home)

The TOL appreciation induced by reshoring policies raises the relative costs of Home exporters while reducing those of Foreign exporters, causing Home exports to decrease and Foreign exports to increase. Figure 6 illustrates that lump-sum subsidies result in the largest reduction in Home exports. Lump-sum subsidies are more destructive to exports than other policies because they more substantially increase labor demand by encouraging the entry of relatively low-productivity reshoring firms, thereby driving up wages significantly. This erodes the price competitiveness of exporters and causes a larger decline in Home exports.

As illustrated in Figure 6, the trade balance improves despite the decline in Home exports. In this paper, the definition of the trade balance incorporates the value added generated by offshore firms. Because reshoring policies reduce the overall volume of offshore production, the offshore value added imported from the Foreign economy decreases substantially following the policy shock. Consequently, by dampening all offshore activities, the reshoring policies in this paper induce a drop in offshore imports that outweighs the reduction in exports, thereby improving the trade balance.

All reshoring policies successfully increase the number of reshoring firms. In particular, tariffs outperform the other instruments in expanding the reshoring extensive margin due to having the lowest wage pressure in the domestic market. Unlike tariffs, both lump-sum and tax credit subsidies significantly bid up domestic wages while reducing offshore firms. Because tariffs operate on foreign production costs, the resulting upward pressure on domestic wages is relatively small. This differential wage effect implies that the two subsidies make domestic entry more difficult, thereby reducing the total number of firms to a greater extent than tariffs. Therefore, under the subsidy regimes, the contraction of offshore firms is largely offset by the exit of domestic firms, failing to significantly expand the reshoring count. In contrast, tariffs achieve the largest expansion in reshoring firms because the domestic wage pressure is relatively muted.

However, tariffs fall short of subsidy policies in terms of qualitatively revitalizing the domestic economy, as they induce a contraction in global demand and severe resource misallocation. Unlike subsidies, tariffs directly and significantly reduce Foreign exports, leading to a sharp decline in Foreign output and income, which subsequently dampens Foreign demand for Home exporters. Furthermore, tariffs severely erode the profitability of offshore firms, shrinking offshore value added and further contracting Home output. In conclusion, while tariffs can quantitatively expand the reshoring extensive margin, they represent a qualitatively inferior reshoring policy that substantially weakens global integration and ultimately contracts Home output.

Tax credit subsidies generate greater reshoring value-added (VA) than alternative policies. The superiority of the tax credit approach in maximizing the reshoring VA/GDP ratio stems from its design, which specifically incentivizes production by more productive firms. Because tax credits are provided in proportion to output, their benefits are concentrated among high-productivity firms; these firms yield a higher VA through economies of scale and marginal cost reductions from intangible investment. In contrast, since lump-sum subsidies provide in identical amounts regardless of productivity, they create a relatively higher incentive for lower-productivity firms to enter, thereby resulting in weaker VA creation across the reshoring firms. Tariffs induce reshoring by increasing foreign costs and reducing foreign profits; since they do not directly enhance domestic profitability, they are less effective than tax credits at increasing reshoring VA.

Lump-sum subsidies drive the steepest wage growth, which ultimately discourages firm entry, leading to a significant reduction in the total number of firms and a subsequent rise in market concentration. As a result, lump-sum subsidies induce a larger decrease in the total number of firms compared to other policy alternatives. While tax credits also lead to wage increases, the reduction in the total number of firms is less severe. This is because tax credits attract more productive firms, which lowers the prices of reshored goods and boosts profits, thereby partially offsetting rising labor costs. Consequently, the decline in firm entry remains less pronounced under tax credits than under lump-sum subsidies. Lastly, tariffs cause the total number of firms to decrease despite falling wages, as the contraction in expected profits outweighs the dampening effect of lower labor costs.

Reshoring policies in an intangible-intensive economy induce a medium-run consumption expansion, whereas tariffs trigger a contraction in output. As illustrated in Figure 6, reshoring subsidies, both lump-sum and tax-credit subsidies, stimulate medium-run consumption and output; notably, lump-sum subsidies yield the most substantial gains by driving wage-induced income growth. Tariffs also boost consumption, primarily because tariff revenues are transferred back to consumers as lump-sum payments, thereby enhancing their purchasing power. Provided there is no foreign retaliation, these gains can be maintained (Ignatenko et al., 2025; Auray et al., 2026). On the production side, reshoring subsidies bolster Home output, whereas tariffs trigger a broad contraction. This domestic output contraction under the tariff regime operates through two main channels: first, it severely erodes export performance due to a loss of price competitiveness following TOL appreciation and a shrinkage in Foreign demand; second, unlike the subsidy regimes, tariffs directly induce a precipitous drop in offshore value added by penalizing foreign production. Meanwhile, foreign output and consumption contract significantly in the medium run, as reshoring policies exert direct downward pressure on foreign labor demand and wages.

In the long run, following the termination of reshoring policies, the cessation of all policy instruments triggers a persistent economic downturn. As the profitability of reshored firms turns negative post-termination, the aggregate average firm value deteriorates. This erosion of firm value induces wealth losses for households, leading to a substantial decline in consumption. Specifically, the expiration of both subsidy regimes shifts production incentives, lowering domestic labor demand while relatively increasing foreign labor demand; this drives down domestic wages and exacerbates the contraction in both consumption and output. Notably, the lump-sum subsidy regime induces a more severe contraction in consumption, output, and wages than the tax credit regime, primarily because it disproportionately incentivized the entry of less productive firms. Because tax credits target relatively more productive firms, fewer firms exit the market when profitability deteriorates post-policy compared to the lump-sum counterpart. Consequently, under the lump-sum subsidy, the post-policy average firm value drops more precipitously, wages collapse further, and both consumption and output plunge. These findings suggest that tax credit subsidies represent a superior policy option to lump-sum subsidies in terms of mitigating long-run economic contractions.

Conversely, the post-policy long-run outcomes under the tariff regime present a notable contrast to its implementation period, where they initially contracted output and wages through the erosion of exports and offshore VA. Once tariffs are lifted, the relative increase in foreign labor demand leads to a depreciation of the TOL. This adjustment successfully rehabilitates export performance and offshore value added, ultimately driving a rebound in domestic output and an improvement in wages. Nevertheless, tariffs fail to sustain consumption in the long run, triggering a post-policy contraction. This consumption decline is primarily attributed to the cessation of the revenue transfer mechanism, as households no longer receive the lump-sum rebates of tariff revenues.

On the other hand, the cessation of all reshoring instruments rehabilitates production profitability in the Foreign country, which in turn stimulates Foreign firm entry and exerts upward pressure on Foreign labor demand and wages. In particular, the expiration of lump-sum subsidies, which causes the sharpest decline in Home wages, induces a relative TOL depreciation for Foreign, thereby driving a significant surge in Foreign consumption and output. Conversely, tariffs, characterized by a smaller post-policy increase in Foreign wages, yield the smallest recovery in Foreign consumption and output.

6 Empirical Evidence

This section evaluates whether the key assumptions and predictions of the model are supported by micro-level U.S. manufacturing data (Compustat Historical Segment) and macro-level industry data (NBER-CES Manufacturing Industry Database). The analysis documents three distinct patterns: first, reshoring subsidies—lump-sum subsidies and tax credits—significantly increase the value added (VA) of intangible-intensive manufacturing industries, whereas tariffs significantly reduce VA in these sectors; second, reshoring subsidies drive a significant rise in personal consumption expenditures (PCE); and third, tax credits exhibit a stronger tendency to induce the reshoring of high-productivity offshore firms compared to alternative policy instruments. While these findings do not necessarily imply a causal relationship, the fact that the correlations implied by the model's results are present in the data is reassuring.

To conduct the empirical analysis, this paper utilizes the NBER-CES Manufacturing Industry Database, which encompasses 473 disaggregated manufacturing industries; the New Industrial Policy Observatory (NIPO), which provides information on industrial policies; and the Compustat Historical Segment Data, which offers microdata on manufacturing firms. The NBER-CES Manufacturing Industry Database is a comprehensive industry-level panel dataset, available from 1958 to 2018, that is well-suited for examining dynamic responses to policies while controlling for industry-specific characteristics. The NIPO data advantageously classifies national industrial policies and identifies targeted products using HS codes, though its time series is limited, starting only in 2009. The Compustat Historical Segment data provides information on manufacturing firms' intangible investments and reshoring activities, covering a time series from 1976 to the present.

This section measures empirical evidence by combining the datasets as follows: We isolate U.S. industrial policies from the NIPO dataset, classify them into lump-sum subsidies, tax credits, and tariffs[7], and map their HS codes to the NAICS codes in the NBER-CES Manufacturing Industry Database. Multiple concurrent policies within an industry are recorded as a higher policy count, representing greater policy intensity. Although using policy counts to proxy intensity—due to NIPO data limitations—obscures the precise intensive margin, this approach remains valuable given the severe scarcity of industrial policy data. Additionally, using Compustat Historical Segment Data, firm-level intangible investment is calculated as the sum of R&D expenses and 30% of Selling, General, and Administrative (SG&A) expenses (Corrado et al., 2006; Eisfeldt and Papanikolaou, 2013). These are then aggregated by NAICS industry and merged into the NBER-CES Manufacturing Industry Database.

6.1 The Effect of Reshoring Policies on Industrial Value Added

To analyze the impact of reshoring policies on the real value added (VA) of U.S. manufacturing industries, the Local Projection (LP) method proposed by Jordà (2005) was utilized, and the estimated equation is presented in equation (32). The dependent variable is the cumulative log change in real VA ($\ln(VA_{t+h}) - \ln(VA_{t-1})$) at horizon h relative to the pre-policy period ($t-1$). The estimation window spans $h \in \{-3, -2, ..., 5\}$, normalized to $h = -1$. To isolate the pure policy effect, industry and year fixed effects are included to absorb common macroeconomic shocks. Lagged log employment, lagged log capital stock, and lagged log total factor productivity are included as explanatory variables to account for pre-policy industry size. Standard errors are clustered at the NAICS industry level. Because this paper evaluates reshoring policy impacts on intangible-intensive industries, the analysis is restricted to the top 25% of industries by intangible investment.

$$\ln(VA_{j,t+h}) - \ln(VA_{j,t-1}) = \beta_{1,h} Policy_{jt} + \Gamma_h \mathbf{X}_{j,t-1} + \alpha_j + \delta_t + \epsilon_{j,t+h} \tag{32}$$

$\ln(VA_{j,t+h}) - \ln(VA_{j,t-1})$: Cumulative growth in real VA for industry j from $t-1$ to $t+h$. $Policy_{jt}$: Number of reshoring policy shocks in year t. $\mathbf{X}_{j,t-1}$: Control variables including log of employment ($\ln(Emp_{j,t-1})$), log of capital stock ($\ln(Cap_{j,t-1})$), log of total factor productivity ($\ln TFP4_{j,t-1}$). α_j, δ_t: Industry and year fixed effects. h: Forecast horizon ($h = 0, \ldots, 5$).

The empirical results show that reshoring subsidies (lump-sum subsidies and tax credits) significantly increase cumulative real VA within the top 25% of intangible-intensive U.S. manufacturing industries, whereas tariffs significantly reduce cumulative real VA. Specifically,

[7] Lump-sum subsidies include financial grants, in-kind grants, loan guarantees, state loans & aid, and equity stakes; Tax Credits include production subsidies, tax-based export incentives, tax or social insurance relief, local VA/labor/content incentives, and financial incentives; Tariffs include import tariffs, import tariff quotas, and other import charges. Following the GTA classification, only 'Red' (harmful) and 'Amber' (potentially harmful) entries are included to capture discriminatory policy shocks.

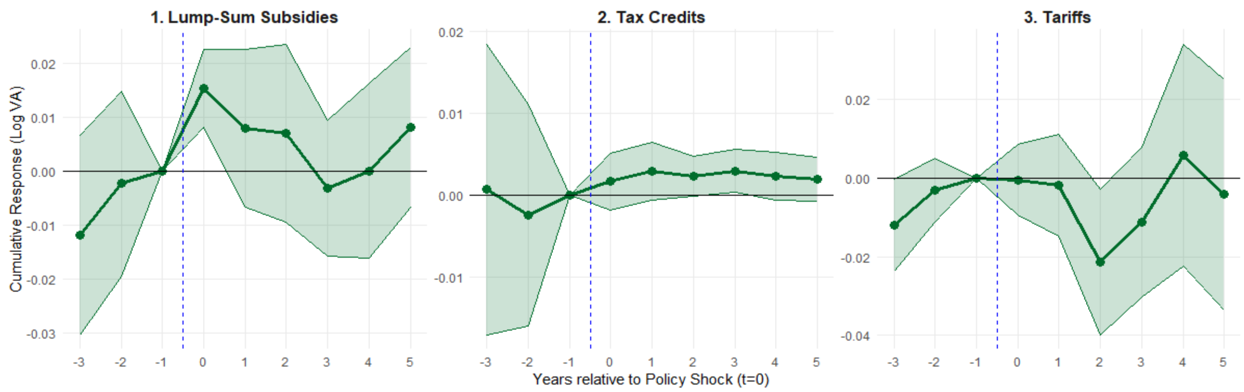

Figure 7: The Effect of Reshoring Policies on Industrial VA
Data: NBER-CES Manufacturing Industry Database, New Industrial Policy Observatory (NIPO), Compustat Historical Segment
Note: Shaded areas represent 95% confidence intervals.

the implementation of lump-sum subsidies led to an immediate and significant increase in VA of approximately 2% at the 95% confidence level. In the case of tax credits, VA increased significantly by about 0.3% after three years. Tariffs resulted in a significant decrease in real VA of approximately 2% two years after policy implementation at the 95% confidence level. This empirical finding aligns precisely with the model results shown in Figure 6, where the two reshoring subsidies improve real output while tariffs cause output contraction.

6.2 The Effect of Reshoring Policies on Personal Consumption Expenditures

To analyze how reshoring policies affected Personal Consumption Expenditures (PCE) linked to U.S. manufacturing industries, I integrated the NBER-CES Manufacturing database, NIPO, and the BEA's real PCE by type of product data.[8] Because the data matches industry-specific rather than total PCE, it does not perfectly align with the aggregate real consumption predicted by the model. However, industry-level PCE was chosen as the dependent variable because aggregate data is limited (available only since 2007), and using sector-specific data allows us to isolate the heterogeneous policy effects on consumption across industries. The BEA PCE data was matched at the industry level to the same dataset used in Section 7.1, and the Local Projection method was employed for the analysis. Explanatory variables include lagged log employment, capital stock, total factor productivity, and wages, along with lagged PCE growth to control for pre-trends. As this paper evaluates the impact of reshoring policies on intangible-intensive industries, the analysis is restricted to the top 25% of industries by intangible investment.

$$\ln(PCE_{j,t+h}) - \ln(PCE_{j,t-1}) = \beta_h Policy_{jt} + \gamma_h \Delta \ln(PCE_{j,t-2}) + \Gamma_h \mathbf{X}_{j,t-1} + \alpha_j + \delta_t + \epsilon_{j,t+h} \tag{33}$$

[8]A PCE bridge was used to map the 402 PCE commodities from BEA industry codes to NAICS codes.

$\ln(PCE_{j,t+h}) - \ln(PCE_{j,t-1})$: Cumulative growth in real Personal Consumption Expenditures from $t-1$ to $t+h$ in industry j. $Policy_{jt}$: Number of reshoring policy shocks in year t. $\Delta \ln(PCE_{j,t-2})$: Pre-intervention trend control ($\ln PCE_{j,t-2} - \ln PCE_{j,t-3}$). $\mathbf{X}_{j,t-1}$: Vector of lagged control variables including $\ln(TFP4_{j,t-1})$, $\ln(Emp_{j,t-1})$, $\ln(Cap_{j,t-1})$, and $\ln(Wage_{j,t-1})$. α_j, δ_t: Industry (NAICS 6-digit) and year fixed effects. h: Forecast horizon ($h = -3, \ldots, 5$, where $h = -1$ is the baseline).

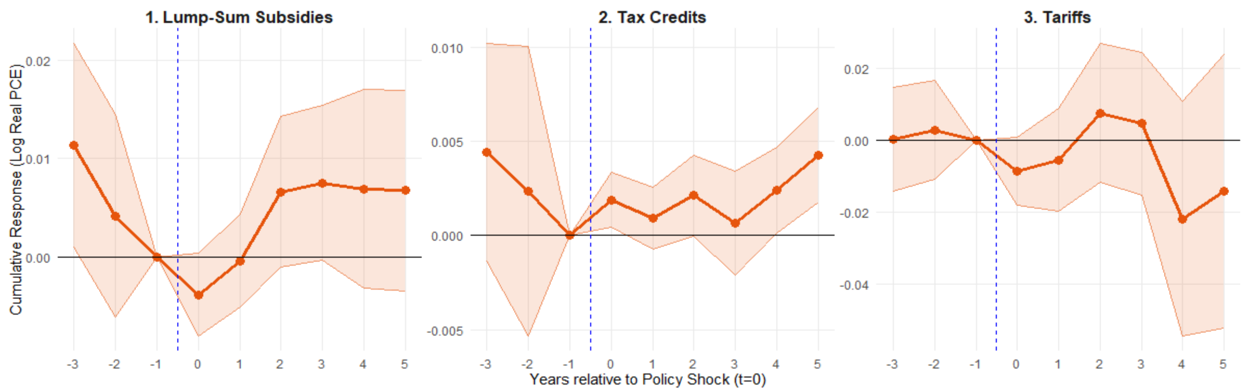

Figure 8: The Effect of Reshoring Policies on Personal Consumption Expenditures
Data: NBER-CES Manufacturing Industry Database, New Industrial Policy Observatory, Compustat Historical Segment, BEA
Note: Shaded areas represent 95% confidence intervals.

The empirical results show that within the top 25% of intangible-intensive U.S. manufacturing, tax credits significantly increase cumulative real PCE by approximately 0.25%. Lump-sum subsidies increase real PCE by 0.5%, though this is only significant after three years at the 90% confidence level. Tariffs decrease cumulative PCE, but the effect is statistically insignificant.

Excluding tariffs, these empirical findings for reshoring subsidies align with the model's prediction in Figure 6 that consumption rises following policy implementation. The ambiguous effect of tariffs stems from scope differences between the data and the model. The theoretical model applies tariffs to both Home offshore firms and Foreign exporters, whereas the empirical data only covers U.S. manufacturing firms, thereby failing to capture the impact of Foreign exporters on PCE. Furthermore, the model assumes no foreign retaliation and a full rebate of tariff revenues to households—ideal conditions that stimulate consumption but are difficult to meet in reality, explaining the discrepancy between the model and the data.

6.3 Average Productivity in Reshoring Policy-Exposed Industries

The model assumes that tax credits reshore more productive firms than other reshoring policies. This assumption aligns with the average productivity patterns derived from industry-level data exposed to actual reshoring. To identify reshored firms, I utilized Compustat Historical Segment Data and defined a firm as "reshored" if it met the following criteria: (i) for firms currently offshoring, the domestic investment share exceeds its preceding 3-year moving average by more than 1.2 standard deviations (σ), and investment in foreign affiliates decreased from the previous period; or (ii) for firms with limited history, the domestic investment share increases by more than 10 percentage points, while foreign affiliate investment decreases.[9] The count of these reshored firms by industry was then integrated into the NBER-CES Manufacturing Industry Database. By matching NIPO data with industry-level productivity (TFP4) from the NBER-CES Manufacturing Industry Database, the average productivity of targeted reshoring industries was calculated.

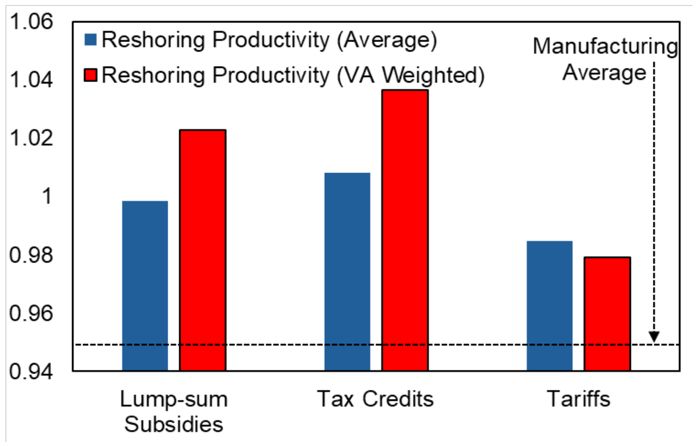

Figure 9: Average Productivity in Reshoring Policy-Exposed Industries
Data: NBER-CES Manufacturing Industry Database, New Industrial Policy Observatory (NIPO), Compustat Historical Segment
Note: Reshoring firms are identified by a domestic investment share exceeding its 3-year moving average by 1.2σ, alongside a decline in foreign investment. For firms with limited history, a ≥ 10 pp increase in domestic share is required.

As shown in Figure 9, among industries with reshored firms, those exposed to tax credits exhibited the highest average productivity, followed by lump-sum subsidies and tariffs. VA-weighted productivity calculations yielded consistent results: tax credits ranked highest, followed by lump-sum subsidies, while industries with tariff-induced reshored firms demonstrated the lowest productivity.

7 Welfare

This section examines how the welfare effects of reshoring policies vary across the Home and Foreign. We define welfare as the lifetime utility derived from consumption. In the absence of reshoring policies, the initial steady-state welfare is $\mathcal{U}_0 \equiv \frac{1}{1-\beta}\left(\frac{C_0^{1-\gamma}}{1-\gamma}\right)$, where C_0 represents the Home household's consumption at the initial equilibrium. Similarly, the lifetime utility at the new steady state after implementing reshoring policies is: $\mathcal{U}_{201}^{RP} \equiv \frac{1}{1-\beta}\left(\frac{C_{201}^{1-\gamma}}{1-\gamma}\right)$, where C_{201} is the Home household's consumption at the post-policy steady state. Then, the welfare with reshoring policies, including transition paths, can be expressed as: $\mathcal{U}_0^{RP} = \sum_{t=0}^{200} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} + \beta^{201}\mathcal{U}_{201}^{RP}$. Foreign welfare is measured in the same manner as that of Home.

The lifetime welfare gain is calculated in initial consumption-equivalent terms (Δ). To compare the dynamics with and without reshoring policies, this paper distinguishes between

[9] Offshore and domestic plants were distinguished using the firms' geographic segments.

the cumulative welfare gain along the transition path (Δ) and the steady-state welfare gain (Δ_{ss}) derived from comparative statics between the initial and new steady states.

$$\Delta = \left(\frac{\mathcal{U}_0^{RP}}{\mathcal{U}_0}\right)^{\frac{1}{1-\gamma}} - 1 \quad \text{and} \quad \Delta_{ss} = \left(\frac{\mathcal{U}_{201}^{RP}}{\mathcal{U}_0}\right)^{\frac{1}{1-\gamma}} - 1 \tag{34}$$

which satisfy $\mathcal{U}_0^{RP} = \frac{1}{1-\beta}\left\{\frac{[(1+\Delta)C_0]^{1-\gamma}}{1-\gamma}\right\}$ and $\mathcal{U}_{201}^{RP} = \frac{1}{1-\beta}\left\{\frac{[(1+\Delta_{ss})C_{201}]^{1-\gamma}}{1-\gamma}\right\}$.

Table 3 presents the welfare effects of reshoring policies for the Home and Foreign. Notably, all reshoring policies in the intangible economy generate transitional welfare gains. Reshoring subsidies yield transitional gains as higher domestic labor demand raises wages and income, bolstering consumption. Tariffs produce transitional gains by increasing consumer purchasing power via tax revenue rebates. Lump-sum subsidies provide the largest transitional welfare gains for Home because they induce reshoring by relatively inefficient firms, driving the sharpest short-term wage growth.

		Δ	Δ_{ss}
Lump-sum Subsidies	Home	0.064	-0.000
	Foreign	-0.055	0.001
Tax Credits	Home	0.012	-0.000
	Foreign	-0.002	0.001
Tariffs	Home	0.002	0.000
	Foreign	-0.002	0.000

Table 3: Welfare changes following reshoring policies (%, initial consumption)

In steady-state comparisons, none of the reshoring policies exhibit significant changes in welfare. This is because the policies are implemented for only 40 periods before being terminated, causing the steady-state consumption at period 201 to remain largely unchanged relative to the pre-policy baseline. However, both lump-sum and tax credit subsidies are found to induce a very marginal steady-state welfare loss in the long run.

In an intangible economy, reshoring policies yield prominent domestic efficiency gains driven by scalability during the transition path, primarily due to the reduction in Foreign production. These policies may be justified if the primary objective is to induce transitional Home welfare gains at the expense of Foreign welfare.

7.1 Ramsey-Optimal Policy

The Ramsey authority maximizes the Home welfare function in equation (1) subject to the equilibrium constraints of the economy, which are detailed in Appendix C. Following standard literature practice, this paper casts the original non-stationary Ramsey problem into a recursive stationary form, and the Ramsey optimal policy is computed under a timeless perspective (Woodford, 2003).

In this section, our focus lies in how the Ramsey planner determines the optimal reshoring policies $(\eta, t_{s,t}, \tau_t)$. Implementing reshoring policies as a form of protectionism introduces distortions by sacrificing the Foreign offshoring cost advantage; however, it concurrently improves domestic production efficiency by repatriating intangible capital to the Home country. That is, the Ramsey authority determines the optimal policy mix by weighing the trade-off between policy-induced distortions and the beneficial scalability effects.

Table 4 reports the welfare gains under the optimal long-run Ramsey policy for each policy instrument. To prevent spurious welfare reversals, this paper assumes identical initial conditions across all policies and incorporates full transition dynamics. Regardless of the reshoring policy implemented after $t = 0$, all state variables at $t = 1$ are set to their no-policy steady-state levels. We then compare lifetime welfare under the optimal reshoring regime with the no-policy baseline, both starting from $t = 0$. The long-run welfare gains of the Ramsey policy are measured as the percentage increase in consumption that makes households indifferent between the regimes with and without reshoring policies.

Policy Scenario	**Welfare Gains**	**Policy Instruments**		
		Lump-sum ($\boldsymbol{\eta}$)	Tax Credits ($\boldsymbol{t_s}$)	Tariffs ($\boldsymbol{\tau}$)
Single Instrument				
Lump-sum Subsidies	1.68%	0.46 [42.1%]	–	–
Tax Credits	0.14%	–	0.077	–
Tariffs	0.24%	–	–	0.134
Policy Mix				
Lump-sum & Tax Credits	1.57%	0.433 [37.4%]	0.044	–
Lump-sum & Tariffs	2.19%	0.450 [37.0%]	–	0.096
Tax Credits & Tariffs	0.35%	–	0.031	0.050

Table 4: Ramsey Optimal Policy Mix

Note: Welfare gains are measured against the counterfactual scenario in which reshoring policies are absent. [] denotes the share of lump-sum subsidies relative to reshoring firms' profits.

Table 4 reports that the welfare gain from the Ramsey-optimal policy mix—comprising lump-sum subsidies and tariffs—reaches approximately 2.19% of annual steady-state consumption. Specifically, Home's gains from intangible reshoring are maximized when a 37.0% lump-sum subsidy (relative to reshoring firms' profits) is implemented alongside a 9.6% tariff. The second-best outcome is achieved by using a lump-sum subsidy as a single instrument, which yields a welfare improvement of 1.68%. By providing a substantial subsidy equivalent to 42.1% of profits, the Home economy significantly bolsters domestic intangible investment, thereby driving the second-highest welfare gains. The Ramsey-optimal policy mix consistently incorporates lump-sum subsidies because they generate the most pronounced wage growth among all reshoring instruments, thereby serving as the primary driver of consumption. Simultaneously, tariffs further enhance consumption and welfare by penalizing foreign-produced varieties and incentivizing the relocation of production to the Home market.

These results suggest that reshoring policies targeting intangible-intensive industries can induce net welfare gains by stimulating domestic intangible investment, even though they erode the cost advantages previously enjoyed through offshore production. The reduction in marginal costs and subsequent profit improvements—driven by the return of intangible scalability from Foreign to Home—ultimately translate into higher consumption and welfare for Home households. Furthermore, contrary to traditional literature, tariffs can enhance welfare despite a reduction in trade when applied appropriately to intangible-intensive sectors. In this context, they foster domestic intangible retention and encourage high-tech manufacturing. Crucially, such interventions must be concentrated on intangible-intensive industries capable of generating scalable benefits and should specifically target high-productivity firms that are currently capturing significant profits from offshore operations.

8 Conclusion

Recent reshoring policies in advanced economies have shifted focus toward securing leadership in intangible-intensive industries transcending traditional manufacturing protection. This study integrates the scalability of intangible capital into a two-country general equilibrium model with endogenous firm entry and exit and offshoring to analyze the macroeconomic effects of these policies. Specifically, the paper examines how the three reshoring policies influence firms' offshoring choices in the presence of intangible capital scalability, leading to structural shifts in aggregate macroeconomic outcomes.

The findings are as follows. First, reshoring policies in intangible-intensive industries generate a transitional expansion in consumption. Reshoring subsidies stimulate the economy by bolstering labor demand and wages, while tariffs increase consumption through revenue transfers to households. In contrast to subsidies, however, tariffs contract domestic output by curtailing exports and offshore value added.

Second, tax credits are found to maximize reshoring value-added because they provide stronger incentives for productive firms to reshore compared to alternative policies. Since tax credits are structured to increase with output, they tend to repatriate high-productivity firms—unlike lump-sum subsidies—thereby expanding value creation.

Third, all reshoring policies targeting intangible-intensive firms induce transitional welfare gains through intangible scalability. Reshoring subsidies expand labor demand, which raises wages, expands consumption, and increases welfare. In particular, because lump-sum subsidies are non-proportional to productivity, they drive a more pronounced increase in wages, leading to the largest improvements in consumption and welfare. Tariffs also yield welfare gains because tariff revenues are fully rebated to households and foreign retaliation is absent.

Finally, the Ramsey optimal policy mix—comprising lump-sum subsidies at 37.0% of reshoring firms' profits and 9.6% tariffs—improves welfare by 2.19%. These results imply that reshoring policies targeting intangible-intensive industries, despite eroding the cost advantages of offshore production, can generate welfare gains by inducing an increase in domestic intangible investment.

In conclusion, this study confirms that reshoring policies in intangible-intensive industries can function as medium-run stimulus mechanisms due to scalability. The theoretical framework established herein serves as a critical foundation for evaluating the welfare implications of future industrial policies targeting high-tech industries.

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Appendix A. Empirical Facts and Event Study

This study utilizes Compustat Segment data to establish stylized facts regarding the impact of intangible capital investment on offshoring decisions and aggregate productivity. These empirical findings provide the rationale for the assumptions adopted in the benchmark model presented in the main text.

The Compustat Segment data covers 28,900 North American firms, representing 67% of the total database. This dataset is instrumental for analyzing the operating structures and geographical segmentation of firms expanding into foreign markets. It is uniquely suited for this study as it not only facilitates the identification of offshore plant locations but also contains the micro-level data—specifically R&D and Selling, General, & Administrative (SG&A) expenses—necessary to construct aggregate measures of intangible investment.

Firms are identified using the "GEOTP" variable within the operating segment type classification. Because this variable details the geographic locations of operations, it allows for a clear distinction between the domestic plants and foreign affiliates of multinational corporations. Following Corrado et al. (2006) and Eisfeldt and Papanikolaou (2013), intangible investment is defined as the sum of R&D expenses and organizational capital investment, the latter calculated as 30% of SG&A expenses. For the purpose of this analysis, total factor productivity (TFP) is approximated as sales per employee, while marginal cost is proxied by the ratio of cost of goods sold (COGS) to sales, assuming COGS represents variable costs. Manufacturing industries are defined by NAICS codes 311–339. Based on this data, I document the following stylized facts:

(i) The scalability of intangible investment (Figure 10): Manufacturing firms demonstrate scalability, characterized by a negative correlation between intangible investment and marginal costs. This finding validates the assumption in the Section 3 benchmark model that intangible investment drives reductions in marginal cost.

(ii) Revenue growth with intangible investment (Figure 11): There is a positive relationship between intangible investment and average sales. As shown in Figure 11, average sales rise distinctively as manufacturing firms progress to higher R&D quantiles. This corroborates the model's implication that intangible capital accumulation contributes to increased firm profitability.

(iii) Productivity sorting into intangible investment and offshoring (Figure 12): More productive firms make higher intangible investments and engage more in offshoring. The benchmark model relies on the assumption that only productive firms can overcome the fixed costs of intangible investment and offshoring. Figure 12 validates this assumption empirically: average productivity clearly increases alongside intangible investment quantiles. Additionally, offshore firms display higher average productivity than their domestic counterparts. These

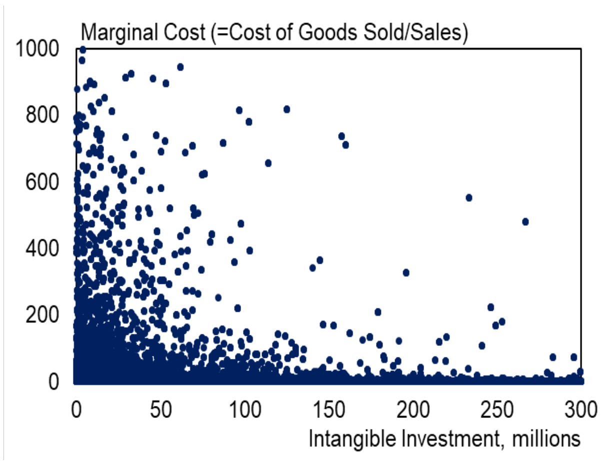


Figure 10: Scalability of Intangible Investment
Source: Compustat Segment

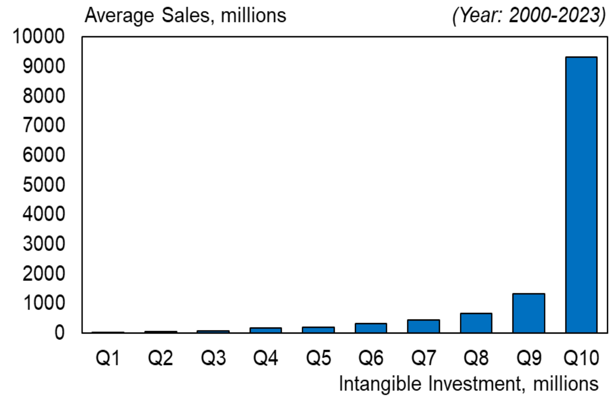


Figure 11: Quantiles of Intangible Investment and Average Sales (Manufacturing)
Source: Compustat Segment

findings provide empirical evidence that high-productivity firms are significantly more likely to undertake both offshoring and intangible investment.

(iv) Offshore firms tend to have higher intangible investment and higher sales compared to non-offshore firms (Figure 13): Figure 13 presents a scatter plot of intangible investment against sales, distinguishing between offshore and domestic entities. The data reveals a significant positive correlation between sales and intangible investment for offshore firms relative to non-offshore firms. This pattern is consistent with the benchmark model, in which offshore firms achieve higher profits through intangible investment and therefore face stronger incentives to invest in intangibles. Based on this empirical fact, I assume—purely for simplicity in the benchmark—that offshore firms necessarily undertake intangible investment.

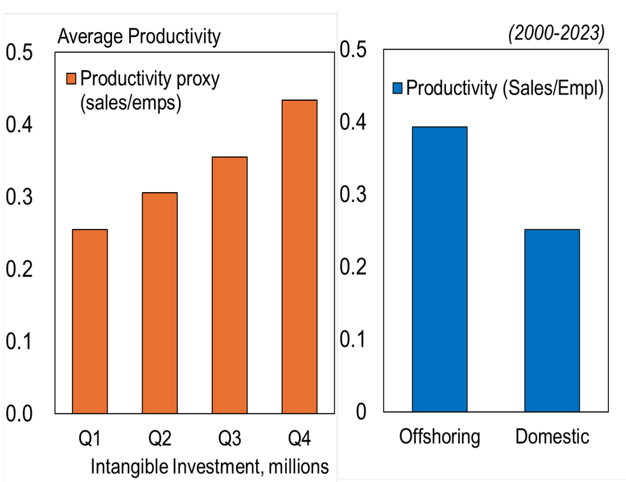


Figure 12: Intangibles Quantile & Productivity, Offshore vs. Domestic Productivity
Source: Compustat Segment

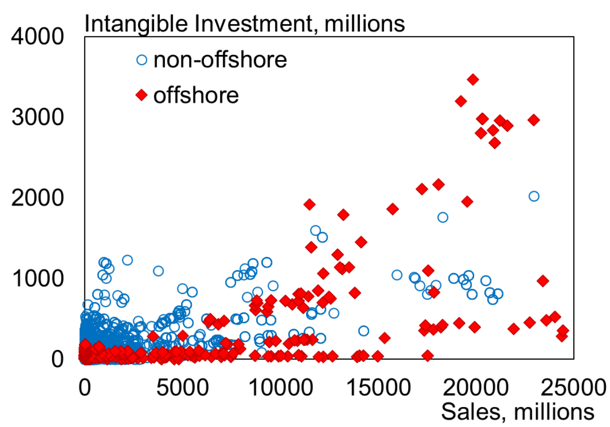


Figure 13: Offshore vs. Non-offshore Intangibles
Source: Compustat Segment
Note: Exclude data for non-offshore firms whose segments are classified as market, division, or product and service.

(v) Positive sales–intangible correlation in high-tech sectors (Figure 14): In manufacturing, intangible investments exhibit a stronger positive correlation with net sales growth within high-tech industries.[10] This observation confirms that among firms with significant intangible assets, technology-intensive firms achieve superior sales growth rates. Consequently, this suggests that the benchmark model is particularly effective for analyzing the dynamics of high-technology manufacturing firms.

(vi) Positive correlation between exports and intangible investment (Figure 15): For manufacturing firms, a rise in intangible investment is associated with increased export activity. In the benchmark model, both exporting and intangible investment entail fixed costs, meaning only firms capable of covering these costs can engage in them. In other words, firms with higher productivity are more likely to export and invest in intangibles. Figure 15 demonstrates this fact, showing that highly productive firms tend to increase both their exports and intangible investments.

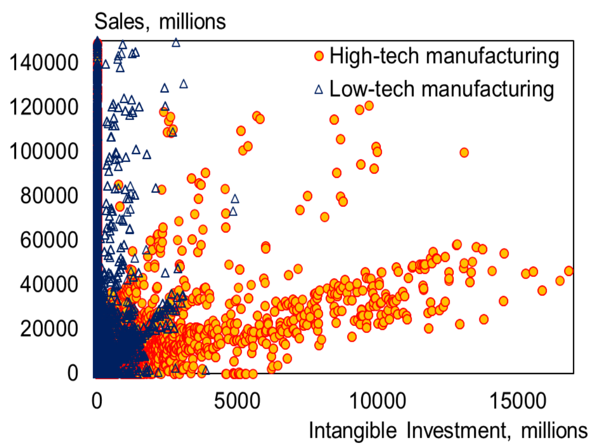


Figure 14: High- and Low-Tech Sales & Intangibles
Source: Compustat Segment

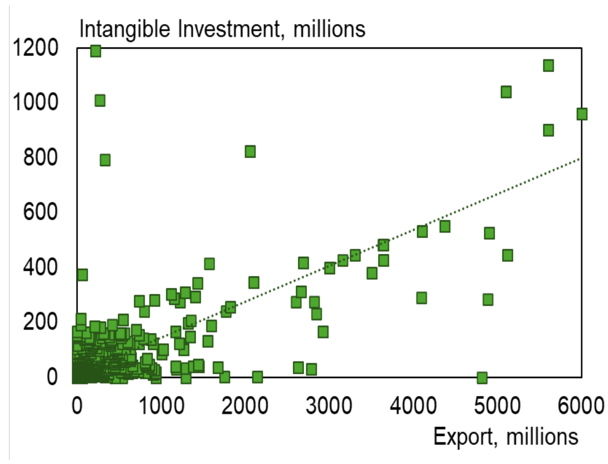


Figure 15: Export and Intangible Investment
Source: Compustat Segment

I executed an Event study using Equation (35) and Compustat Segment data to investigate the impact of intangible capital on sales, exports, offshore value-added, and productivity. To investigate how the introduction of intangible investment affects the decision-making of offshoring firms, I defined the event as a firm's R&D-to-sales ratio exceeding 10% compared to the previous year ($\mathrm{D}_{i,t+j} = 1$ if firm i's R&D-to-sales ratio $\geq 10\%$ at relative year τ). To account for highly productive firms that already conduct intangible investment, I chose not to define the event as a transition from zero R&D investment to positive R&D. Relative year $j = 0$ corresponds to the event year when the R&D-to-sales ratio first exceeds 10%, and

[10]High-tech manufacturing includes industries classified under NAICS codes 325 (Chemicals), 334 (Computer and Electronic Products), 336 (Transportation Equipment), 333 (Machinery), 335 (Electrical Equipment), and 339 (Miscellaneous Manufacturing).

$j = -1$ is used as the omitted reference period. The unit of the event window is years.

$$y_{i,t} = \sum_{j=-5}^{7} \gamma_j D_{i,t+j} + \delta_i + \delta_t + \beta X_{i,t} + \epsilon_{i,t}, \tag{35}$$

$y_{i,t}$ represents sales, offshore value-added (VA), marginal cost, exports, and productivity, and γ_j captures the effect at relative year j. δ_i and δ_t capture firm and year fixed effects, respectively, and $X_{i,t}$ represents control variables.

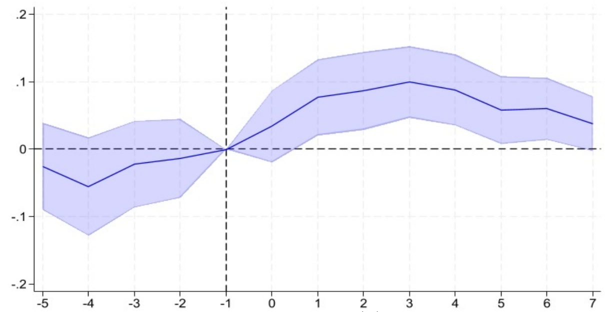
The event study was conducted on manufacturing firms and, specifically, on offshore firms within that group. First, the Intangible investment of manufacturing firms was found to significantly increase net sales, export, and productivity at the 95% confidence level, relative to the control group, which experienced a comparatively smaller increase in R&D investment. The investment also significantly decreased marginal cost, thereby demonstrating a scalable nature. Specifically, as shown in Figure 16, the increase in intangible investment led to a significant rise in manufacturing firms' net sales starting from one year post-event (Figure 16 (a)), and a significant immediate decrease in marginal cost for two years following the event (Figure 16 (b)). Furthermore, export significantly increased for up to two years post-event following the rise in intangible investment (Figure 16 (c)), and productivity significantly increased starting from the fourth year post-event (Figure 16 (d)).

For offshore firms, the increase in intangible investment significantly increased offshore VA (Value Added) and also significantly reduced marginal cost. Offshore VA showed a significant increase over seven years post-event (Figure 16 (e)), and marginal cost significantly decreased for two years post-event, demonstrating scalability (Figure 16 (f)). Conversely, the degree of significant increase in offshore firms' exports and productivity was weaker compared to the overall manufacturing firms. Nevertheless, export significantly increased in the first year post-event following the increase in offshore firms' intangible investment (Figure 16 (g)), and offshore productivity also showed a significant increase starting from the sixth year (Figure 16 (h)).

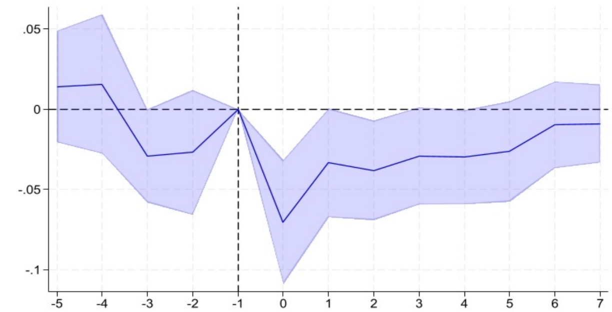
The detailed table of the Event study estimation results can be found in Appendix A. Based on these results, we confirm, using micro-level evidence, that an increase in intangible investment significantly boosts the sales, export, and productivity of both manufacturing and offshore firms, while significantly lowering their marginal cost. This finding implies that the assumptions in the benchmark model—that firms with intangible investment experience increased profit and lower marginal cost—are supported by empirical evidence. Furthermore, the model's foundation, which posits that firms with intangible investment expand their exports and have higher productivity, is also corroborated by this event study.

Figure 16: Manufacturing Firms & Offshore Firms

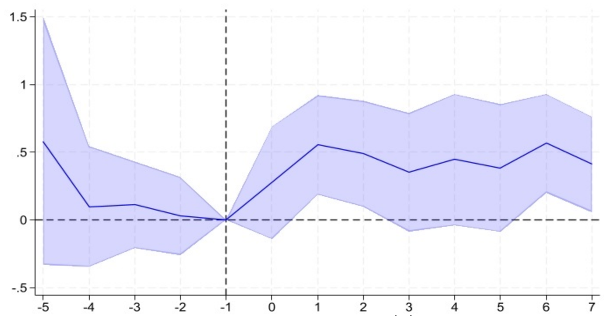
(a) Net Sales



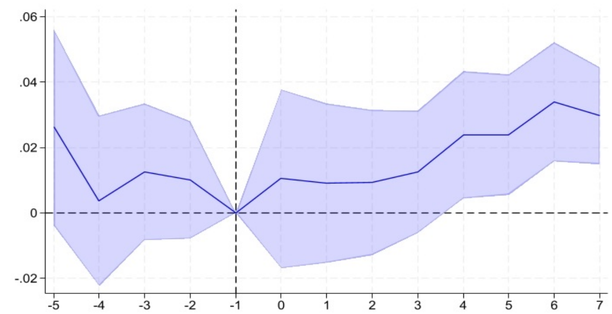
(b) Marginal Costs (Cost of Goods Sold/Sales)



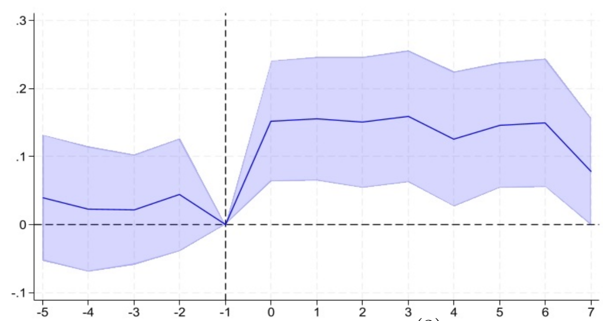
(c) Export$^{(1)}$



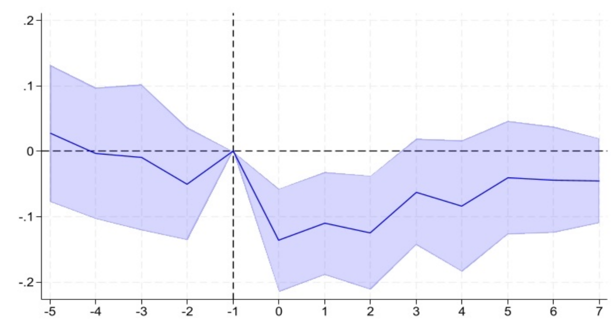
(d) Productivity



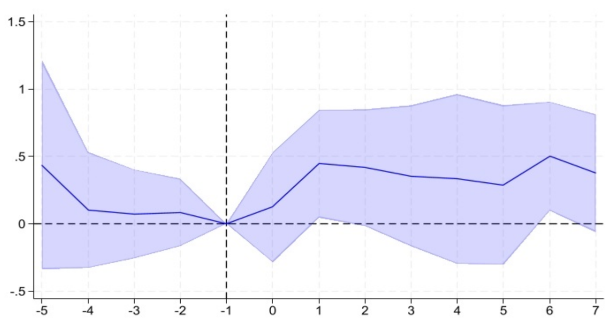
(e) Offshore VA$^{(2)}$



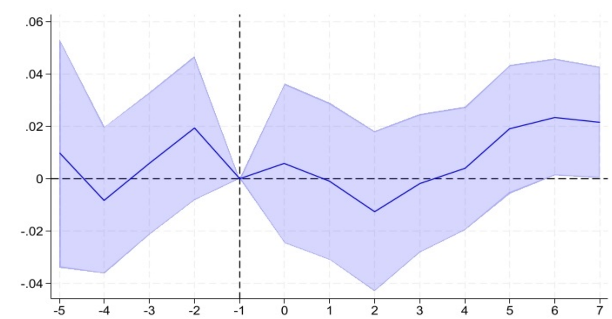
(f) Offshore Marginal Costs



(g) Offshore Export$^{(3)}$



(h) Offshore Productivity



Source: Compustat Segment

Note: Sample period is 2000–2023. For the dependent variables—net sales, offshore VA, export, marginal costs, and productivity measures—the inverse hyperbolic sine (IHS) transformation ($\text{asinh}(x) = \ln(x + \sqrt{x^2 + 1})$) was applied. (1) Missing values for exports are removed. (2) Offshoring VA is measured using sales of vertical offshore plants. (3) Missing values for exports are removed.

.Table 4: Event Study Results

	Manufacturing Firms							
Dependent Variables	Log(Sales)	Log(Marg. Costs)	Log(Export)	Log(Productivity)	Log(Offsh. VA)	Log(Offsh. MC)	Log(Offsh. Export)	Log(Offsh. Prod.)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
4 years before	-0.025	0.014	0.583	0.026	0.040	0.028	0.439	0.010
	(0.033)	(0.018)	(0.464)	(0.015)	(0.047)	(0.053)	(0.394)	(0.022)
3 years before	-0.055	0.016	0.099	0.004	0.023	-0.003	0.104	-0.008
	(0.037)	(0.022)	(0.227)	(0.013)	(0.047)	(0.051)	(0.218)	(0.014)
2 years before	-0.022	-0.029	0.112	0.013	0.022	-0.009	0.074	0.006
	(0.033)	(0.015)	(0.162)	(0.011)	(0.041)	(0.057)	(0.168)	(0.014)
1 year before	-0.014	-0.027	0.029	0.010	0.044	-0.050	0.086	0.019
	(0.030)	(0.020)	(0.146)	(0.009)	(0.042)	(0.044)	(0.127)	(0.014)
Event	0.034	-0.070***	0.276	0.010	0.152***	-0.136***	0.124	0.006
	(0.027)	(0.020)	(0.212)	(0.014)	(0.045)	(0.040)	(0.208)	(0.016)
1 year after	0.077***	-0.033*	0.556***	0.009	0.155***	-0.111***	0.447**	-0.001
	(0.029)	(0.017)	(0.187)	(0.012)	(0.046)	(0.040)	(0.203)	(0.015)
2 years after	0.087***	-0.038**	0.489**	0.009	0.150***	-0.125***	0.417*	-0.012
	(0.029)	(0.016)	(0.199)	(0.011)	(0.049)	(0.045)	(0.220)	(0.016)
3 years after	0.100***	-0.029*	0.353	0.013	0.159***	-0.062	0.356	-0.002
	(0.027)	(0.015)	(0.223)	(0.010)	(0.049)	(0.041)	(0.266)	(0.014)
4 years after	0.088***	-0.030**	0.446*	0.024**	0.126**	-0.084	0.334	0.004
	(0.027)	(0.015)	(0.247)	(0.010)	(0.051)	(0.051)	(0.320)	(0.012)
5 years after	0.058**	-0.026*	0.385	0.024**	0.146***	-0.041	0.291	0.019
	(0.026)	(0.016)	(0.239)	(0.009)	(0.047)	(0.044)	(0.301)	(0.013)
6 years after	0.060**	-0.010	0.566***	0.034***	0.150***	-0.044	0.503**	0.024**
	(0.023)	(0.014)	(0.186)	(0.009)	(0.048)	(0.041)	(0.206)	(0.011)
7 years after	0.038*	-0.009	0.411**	0.030***	0.078*	-0.045	0.376*	0.022**
	(0.021)	(0.012)	(0.179)	(0.008)	(0.040)	(0.033)	(0.222)	(0.011)
Pre-trend	No	Yes	Yes	Yes	No	No	Yes	No
Firm lin. time trends	Yes	No	No	Yes	No	No	No	Yes
FE (year)	No	No	Yes	No	Yes	Yes	Yes	No
FE (firm)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Mean dep. Var.	4.6886	0.8716	3.0913	0.3459	5.0892	0.9215	3.0651	0.3766
# Cluster (firm)	4,307	4,421	413	4,525	1,404	1,403	360	1,573
N	95,996	100,763	5,172	127,255	21,823	21,617	4,449	30,247
Prob > F	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
R square	0.9696	0.8508	0.9457	0.6242	0.9599	0.7361	0.9499	0.8301

Notes: Robust standard errors in parentheses, clustered at the firm level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Dependent variables (export, offshore VA, productivity) are inverse hyperbolic sine transformed. Event study coefficients are relative to year -1 (normalized to 0). All regressions include firm fixed effects.

Appendix B. Existence and Uniqueness of Equilibrium Productivity Cutoffs

The intangible investment cutoff $(z^I_{D,t})$ In equilibrium, the existence of intangible investment productivity cutoff $z^I_{D,t}$ requires lower price for domestically producing firms with intangible investment compared to the domestic firms without intangible investment. To clarify this, I rewrite the per-period profits from domestically producing firms with intangible investment and that of without intangible investment: $d^I_D(z^I_{D,t}) = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t z^I_{D,t}}\right)^{1-\theta} C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)$ and $d^{NI}_D(z^I_{D,t}) = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t z^I_{D,t}}\right)^{1-\theta} C_t$.

Figure 20 plots the three profit functions against the idiosyncratic productivity parameter $z^{\theta-1}$ over the support interval $[z_{min}, \infty)$. Domestic production without intangible investment incurs no fixed costs, resulting in a zero vertical intercept. Domestic production with intangible investment incurs intangible fixed costs, yielding a vertical intercept of $-\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)$. Offshoring incurs both intangible and offshoring fixed costs, resulting in a vertical intercept of $-f_V\frac{w^*_t Q_t}{Z^*_t} - \phi_{V,t}\frac{w^*_t Q_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1)$.

The model predicts that the productivity cutoff $z^I_{D,t}$ exists in equilibrium only if the slope of the profit function for domestic production with intangible investment exceeds that of production without it: $slope\{d^I_{D,t}(z)\} > slope\{d^{NI}_{D,t}(z)\}$. When this condition holds, domestic production with intangible investment yields higher profits for firms with productivity above $z^I_{D,t}$ compared to domestic production without intangible investments. The slope inequality is equivalent to the condition $\psi^{\xi(1-\theta)}_{D,t} > 1$. Since $\psi_{D,t} < 1$, $\xi > 0$, and $\theta > 1$, this condition always holds. It implies that domestic firms investing in intangibles can charge lower prices than those without, ensuring a unique intangible investment cutoff. To avoid a corner solution, profits for domestic firms without intangible investment must exceed those with intangible investment at z_{min} : $d^{NI}_{D,t}(z_{min}) > d^I_{D,t}(z_{min})$.

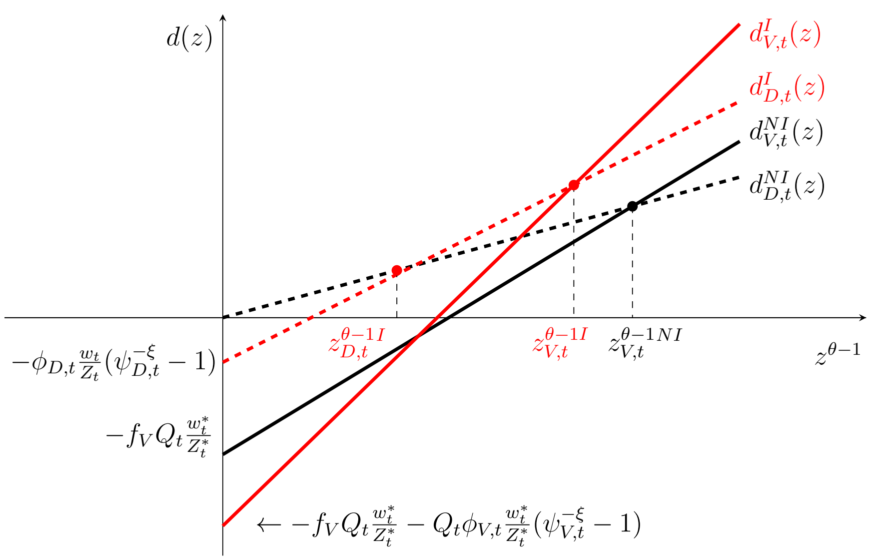


Figure 17: The productivity cutoff

The offshoring cutoff $(z_{V,t})$ In equilibrium, the existence of productivity cutoff $z_{V,t}$ requires a cross-country disparity in effective labor costs, ensuring that a subset of domestic firms find offshore production profitable. To illustrate this, I express per-period profits as: $d^I_{D,t}(z) = M_t\left(\psi^{\xi}_{D,t}\frac{w_t}{Z_t}\right)^{1-\theta} z^{\theta-1} - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1)$ and $d_{V,t}(z) = M_t\left(\tau\psi^{\xi}_{V,t}\frac{w^*_t Q_t}{Z^*_t}\right)^{1-\theta} z^{\theta-1} - f_V\frac{w^*_t Q_t}{Z^*_t} - \phi_{V,t}\frac{w^*_t Q_t}{Z^*_t}(\psi^{-\xi}_{V,t} - 1)$, where $M_t = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\right)^{1-\theta} C_t$ measures the size of Home market.

The model predicts that a productivity cutoff $z_{V,t}$ exists in equilibrium only if the slope of the offshoring profit function exceeds that of domestic production with intangible investment: $slope\{d_{V,t}(z)\} > slope\{d^I_{D,t}(z)\}$. This condition implies that offshoring is more profitable for firms with productivity above $z_{V,t}$.

The slope inequality is equivalent to the condition $\tau(\psi_{V,t}/\psi_{D,t})^{\xi} TOL_t < 1$, where $TOL_t = \frac{Q_t w^*/Z^*_t}{w_t/Z_t}$ denotes the relative cost of effective labor between the foreign and home country in the same currency. This condition requires the effective foreign wage, adjusted for relative marginal cost reductions, to be sufficiently low to offset fixed and iceberg trade costs ($\tau > 1$), incentivizing some domestic intangible investing firms to offshore. To avoid a corner solution, profits for domestic producers with intangible investment must exceed those of offshoring firms at z_{min} ($d^I_{D,t}(z_{min}) > d_{V,t}(z_{min})$). The model's calibration and the scale of macroeconomic shocks guarantee that this condition holds true in all periods.

Appendix C. Model Summary (Financial Autarky)

Average profit (Home)	$\tilde{d}_{D,t}^{I}(z) = \frac{1}{\theta}\tilde{\rho}_{D,t}^{I}(z)^{1-\theta}C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1)$ $\tilde{d}_{D,t}^{NI}(z) = \frac{1}{\theta}\tilde{\rho}_{D,t}^{NI}(z)^{1-\theta}C_t$ $\tilde{d}_{V,t}(z) = \frac{1}{\theta}\tilde{\rho}_{V,t}(z)^{1-\theta}C_t - f_V Q_t \frac{w_t^*}{Z_t^*} - Q_t\phi_{V,t}\frac{w_t^*}{Z_t^*}(\psi_{V,t}^{-\xi} - 1)$ $\tilde{d}_{X,t}(z) = \frac{1}{\theta}\tilde{\rho}_{X,t}(z)^{1-\theta}C_t^* Q_t - f_X\frac{w_t}{Z_t}$
Average profit (Foreign)	$\tilde{d}_{D,t}^{I*}(z^*) = \frac{1}{\theta}\tilde{\rho}_{D,t}^{*I}(z^*)^{1-\theta}C_t^* - \phi_{D,t}^*\frac{w_t^*}{Z_t^*}(\psi_{D,t}^{*-\xi^*} - 1)$ $\tilde{d}_{D,t}^{NI*}(z^*) = \frac{1}{\theta}\tilde{\rho}_{D,t}^{*NI}(z^*)^{1-\theta}C_t^*$ $\tilde{d}_{X,t}^*(z^*) = \frac{1}{\theta}\tilde{\rho}_{X,t}^*(z^*)^{1-\theta}C_t Q_t^{-1} - f_X^*\frac{w_t^*}{Z_t^*}$
Average price (Home)	$\tilde{\rho}_{D,t}^{I}(z) = \frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t\tilde{z}_{D,t}^{I}}$ $\tilde{\rho}_{D,t}^{NI}(z) = \frac{\theta}{\theta-1}\frac{w_t}{Z_t\tilde{z}_{D,t}^{NI}}$ $\tilde{\rho}_{V,t}(z) = \frac{\theta}{\theta-1}\tau Q_t\psi_{V,t}^{\xi}\frac{w_t^*}{Z_t^*\tilde{z}_{V,t}}$ $\tilde{\rho}_{X,t}(z) = \frac{\theta}{\theta-1}\tau^* Q_t^{-1}\frac{w_t}{Z_t\tilde{z}_{X,t}}$
Average price (Foreign)	$\tilde{\rho}_{D,t}^{I*}(z^*) = \frac{\theta}{\theta-1}\psi_{D,t}^{*\xi^*}\frac{w_t^*}{Z_t^*\tilde{z}_{D,t}^{I*}}$ $\tilde{\rho}_{D,t}^{NI*}(z^*) = \frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^*\tilde{z}_{D,t}^{NI*}}$ $\tilde{\rho}_{X,t}^*(z^*) = \frac{\theta}{\theta-1}\tau Q_t\frac{w_t^*}{Z_t^*\tilde{z}_{X,t}^*}$
Free entry	$\tilde{v}_t = \frac{w_t}{Z_t}f_{E,t}$ $\tilde{v}_t^* = \frac{w_t^*}{Z_t^*}f_{E,t}^*$
Total number of firms	$N_t = N_{D,t}^{I} + N_{D,t}^{NI} + N_{V,t}$ $N_t^* = N_{D,t}^{I*} + N_{D,t}^{NI*}$
Law of motion, firms	$N_{t+1} = (1-\delta)(N_t + N_{E,t})$ $N_{t+1}^* = (1-\delta)(N_t^* + N_{E,t}^*)$
Aggregate accounting	$C_t + N_{E,t}\tilde{v}_t(z) = w_t + N_t\tilde{d}_t(z)$ $C_t^* + N_{E,t}^*\tilde{v}_t^*(z^*) = w_t^* + N_{D,t}^*\tilde{d}_t^*(z^*)$
Consumption price index	$1 = N_{D,t}^{I}(\tilde{\rho}_{D,t}^{I}(z))^{1-\theta} + N_{D,t}^{NI}(\tilde{\rho}_{D,t}^{NI}(z))^{1-\theta} + N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta} + N_{X,t}^*(\tilde{\rho}_{X,t}^*(z))^{1-\theta}$ $1 = N_{D,t}^{I*}(\tilde{\rho}_{D,t}^{I*}(z^*))^{1-\theta} + N_{D,t}^{NI*}(\tilde{\rho}_{D,t}^{NI*}(z^*))^{1-\theta} + N_{X,t}(\tilde{\rho}_{X,t}(z^*))^{1-\theta}$
Total profits	$N_t\tilde{d}_t(z) = N_{D,t}^{I}\tilde{d}_{D,t}^{I}(z) + N_{D,t}^{NI}\tilde{d}_{D,t}^{NI}(z) + N_{V,t}\tilde{d}_{V,t}(z) + N_{X,t}\tilde{d}_{X,t}(z)$ $N_t^*\tilde{d}_t(z^*) = N_{D,t}^{I*}\tilde{d}_{D,t}^{I*}(z^*) + N_{D,t}^{NI*}\tilde{d}_{D,t}^{NI*}(z^*) + N_{X,t}^*\tilde{d}_{X,t}^*(z^*)$
Euler equations (Bonds)	$C_t^{-\gamma} = \beta(1 + r_{t+1})E_t(C_{t+1})^{-\gamma}$ $C_t^{*-\gamma} = \beta(1 + r_{t+1}^*)E_t(C_{t+1}^*)^{-\gamma}$

Euler equations (Shares)	$\tilde{v}_t = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\tilde{v}_{t+1}+\tilde{d}_{t+1})\right]$ $\tilde{v}_t{}^* = \beta^*(1-\delta^*)E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}(\tilde{v}^*_{t+1}+\tilde{d}^*_{t+1})\right]$
Average productivity	$\tilde{z}^I_{D,t} = \nu z_{min} z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^k_{V,t} - z^k_{min}}\right]^{\frac{1}{\theta-1}}$ $\tilde{z}^{NI}_{D,t} = \nu z_{min} z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^k_{V,t} - z^k_{min}}\right]^{\frac{1}{\theta-1}}$ $\tilde{z}_{V,t} = \nu z_{min}\left(\frac{N_t}{N_{V,t}}\right)^{1/k}$ $\tilde{z}_{X,t} = \nu z_{min}\left(\frac{N_t}{N_{X,t}}\right)^{1/k}$ $\tilde{z}^{I*}_{D,t} = \nu z^*_{min}\left(\frac{N^*_t}{N^{I*}_{D,t}}\right)^{1/k}$ $\tilde{z}^*_{X,t} = \nu z^*_{min}\left(\frac{N^*_t}{N^*_{X,t}}\right)^{1/k}$
Balanced trade	$N_{X,t}(\tilde{\rho}_{X,t}(z))^{1-\theta}C^*_tQ_t + N_{V,t}\tilde{d}_{V,t}(z) = N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta}C^*_tQ_t + N^*_{X,t}(\tilde{\rho}^*_{X,t}(z^*))^{1-\theta}C_t$

Appendix D. Average Productivity

D.1 The Average Productivity of Offshoring Firms

The average productivity of Home firms that produce offshore is obtained by integrating over the range of the support interval above the offshoring productivity cutoff:

$$\tilde{z}_{V,t} = \left[\frac{1}{1-G(z_{V,t})}\int_{z_{V,t}}^{\infty} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}} = \left[\left(\frac{z_{V,t}}{z_{min}}\right)^{k}\int_{z_{V,t}}^{\infty} z^{\theta-1}\frac{kz_{min}^{k}}{z^{k+1}}dz\right]^{\frac{1}{\theta-1}}$$
$$= \left[\left(\frac{z_{V,t}}{z_{min}}\right)^{k}\frac{kz_{min}^{k}}{k-(\theta-1)}z_{V,t}^{\theta-1-k}\right]^{\frac{1}{\theta-1}} = \nu z_{V,t}, \text{ where } \nu \equiv \left[\frac{k}{k-(\theta-1)}\right]^{\frac{1}{\theta-1}}$$

D.2 The Average Productivity of Domestic Firms

The average productivity of Home firms that produce domestically with intangible investment is obtained by integrating over the range of the support interval above the intangible productivity cutoff and below the offshoring productivity cutoff:

$$\tilde{z}_{D,t}^{I} = \left[\frac{1}{G(z_{V,t})-G(z_{D,t}^{I})}\int_{z_{D,t}^{I}}^{z_{V,t}} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}} = \left[\frac{(z_{D,t}^{I}z_{V,t})^{k}}{z_{min}^{k}(z_{V,t}^{k}-z_{D,t}^{I\,k})}\int_{z_{D,t}^{I}}^{z_{V,t}} z^{\theta-1}\frac{kz_{min}^{k}}{z^{k+1}}dz\right]^{\frac{1}{\theta-1}}$$
$$= \left[\frac{(z_{D,t}^{I}z_{V,t})^{k}}{z_{V,t}^{k}-z_{D,t}^{I\,k}}\frac{k}{k-(\theta-1)}(z_{D,t}^{I\,\theta-1-k}-z_{V,t}^{\theta-1-k})\right]^{\frac{1}{\theta-1}} = \nu\left[\frac{(z_{D,t}^{I}z_{V,t})^{k}}{z_{V,t}^{k}-z_{D,t}^{I\,k}}\left(\frac{1}{z_{D,t}^{I\,k-(\theta-1)}}-\frac{1}{z_{V,t}^{k-(\theta-1)}}\right)\right]^{\frac{1}{\theta-1}}$$
$$= \nu z_{D,t}^{I}z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)}-z_{D,t}^{I\,k-(\theta-1)}}{z_{V,t}^{k}-z_{D,t}^{I\,k}}\right]^{\frac{1}{\theta-1}}$$

The average productivity of Home firms that produce domestically without intangible investment is obtained by integrating over the range of the support interval above the minimum level of the productivity and below the intangible productivity cutoff:

$$\tilde{z}_{D,t}^{NI} = \left[\frac{1}{G(z_{D,t}^{I})}\int_{z_{min}}^{z_{D,t}^{I}} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}} = \left[\frac{z_{D,t}^{Ik}}{z_{D,t}^{I\,k}-z_{min}^{k}}\int_{z_{min}}^{z_{D,t}^{I}} z^{\theta-1}\frac{kz_{min}^{k}}{z^{k+1}}dz\right]^{\frac{1}{\theta-1}}$$
$$= \left[\frac{z_{D,t}^{Ik}}{z_{D,t}^{I\,k}-z_{min}^{k}}\frac{kz_{min}^{k}}{k-(\theta-1)}(z_{min}^{\theta-1-k}-z_{D,t}^{I\theta-1-k})\right]^{\frac{1}{\theta-1}} = \nu\left[\frac{(z_{min}z_{D,t}^{I})^{k}}{z_{D,t}^{I\,k}-z_{min}^{k}}\left(\frac{1}{z_{min}^{k-(\theta-1)}}-\frac{1}{z_{D,t}^{I\,k-(\theta-1)}}\right)\right]^{\frac{1}{\theta-1}}$$
$$= \nu z_{min}z_{D,t}^{I}\left[\frac{z_{D,t}^{I\,k-(\theta-1)}-z_{min}^{k-(\theta-1)}}{z_{D,t}^{I\,k}-z_{min}^{k}}\right]^{\frac{1}{\theta-1}}$$

Appendix E. Average Profits from Domestic Production with and without Intangible Investment, and from Offshore Production

E.1 The relationship between the average profit of offshoring firms and the average profit of domestic intangible firms

The average profit of Home firms that produce domestically with intangible investment is:

$$
\begin{aligned}
\tilde{d}^I_{D,t} = d^I_{D,t}(\tilde{z}^I_{D,t}) &= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t\tilde{z}^I_{D,t}}\right]^{1-\theta} C_t - \phi_{D.t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t}\right]^{1-\theta} C_t\tilde{z}^{I\theta-1}_{D,t} - \phi_{D.t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t}\right]^{1-\theta} C_t(\nu z^I_{D,t}z_{V,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t} - z^{Ik-(\theta-1)}_{D,t}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t z_{V,t}}\right]^{1-\theta} C_t(\nu z^I_{D,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t} - z^{Ik-(\theta-1)}_{D,t}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= d^I_{D,t}(z_{V,t})(\nu z^I_{D,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t} - z^{Ik-(\theta-1)}_{D,t}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)
\end{aligned}
$$

The average profit of the Foreign firms producing offshore is:

$$
\begin{aligned}
\tilde{d}_{V,t} = d_{V,t}(\tilde{z}_{V,t}) &= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{V,t}\tau Q_t\frac{w^*_t}{Z^*_t\tilde{z}_{V,t}}\right]^{1-\theta} C_t - f_V Q_t\frac{w^*_t}{Z^*_t} - \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{V,t}\tau Q_t\frac{w^*_t}{Z^*_t}\right]^{1-\theta} C_t\tilde{z}^{\theta-1}_{V,t} - f_V Q_t\frac{w^*_t}{Z^*_t} - \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1) \\
&= \left\{\frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{V,t}\tau Q_t\frac{w^*_t}{Z^*_t z_{V,t}}\right]^{1-\theta} C_t - f_V Q_t\frac{w^*_t}{Z^*_t} - \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1)\right\}\nu^{\theta-1} \\
&\quad + (\nu^{\theta-1}-1)\left\{f_V Q_t\frac{w^*_t}{Z^*_t} + \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1)\right\} \\
&= \frac{k}{k-(\theta-1)}d_{V,t}(z_{V,t}) + \frac{\theta-1}{k-(\theta-1)}\left\{f_V Q_t\frac{w^*_t}{Z^*_t} + \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1)\right\}
\end{aligned}
$$

The Home firm with productivity equal to the cutoff $z_{V,t}$ is indifferent between producing domestically or offshore. I use the equality of profits at the productivity cutoff, $d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t})$:

$$\begin{aligned}
\tilde{d}_{V,t} &= \frac{k}{k-(\theta-1)} d_{V,t}(z_{V,t}) + \frac{\theta-1}{k-(\theta-1)} \left\{ f_V Q_t \frac{w_t^*}{Z_t^*} + \phi_{V,t} Q_t \frac{w_t^*}{Z_t^*} (\psi_{V,t}^{-\xi} - 1) \right\} \\
&= {z_{D,t}^{I}}^{1-\theta} \left[\frac{z_{V,t}^{k-(\theta-1)} - {z_{D,t}^{I}}^{k-(\theta-1)}}{z_{V,t}^{k} - {z_{D,t}^{I}}^{k}} \right]^{-1} \left\{ \tilde{d}_{D,t}^{I}(z_{V,t}) + \left(1 - (\nu z_D^I)^{\theta-1} \left[\frac{z_{V,t}^{k-(\theta-1)} - {z_{D,t}^{I}}^{k-(\theta-1)}}{z_{V,t}^{k} - {z_{D,t}^{I}}^{k}} \right] \right) \right. \\
&\qquad \left. \phi_{D,t} \frac{w_t}{Z_t} (\psi_{D,t}^{-\xi} - 1) \right\} + \frac{\theta-1}{k-(\theta-1)} \left\{ f_V Q_t \frac{w_t^*}{Z_t^*} + \phi_{V,t} Q_t \frac{w_t^*}{Z_t^*} (\psi_{V,t}^{-\xi} - 1) \right\} \\
&= \left(\frac{z_{V,t}}{\tilde{z}_{D,t}^{I}} \right)^{\theta-1} \left\{ \tilde{d}_{D,t}^{I}(z_{V,t}) + \phi_{D,t} \frac{w_t}{Z_t} (\psi_{D,t}^{-\xi} - 1) \right\} - \frac{k}{k-(\theta-1)} \phi_{D,t} \frac{w_t}{Z_t} (\psi_{D,t}^{-\xi} - 1) \\
&\quad + \frac{\theta-1}{k-(\theta-1)} \left\{ f_V Q_t \frac{w_t^*}{Z_t^*} + \phi_{V,t} Q_t \frac{w_t^*}{Z_t^*} (\psi_{V,t}^{-\xi} - 1) \right\}
\end{aligned}$$

The calculation process utilizes the domestic intangible firms' average productivity equation derived in Appendix D.

$$\tilde{z}_{D,t}^{I} = \nu z_{D,t}^{I} z_{V,t} \left[\frac{z_{V,t}^{k-(\theta-1)} - {z_{D,t}^{I}}^{k-(\theta-1)}}{z_{V,t}^{k} - {z_{D,t}^{I}}^{k}} \right]^{\frac{1}{\theta-1}}$$

$$\Rightarrow {z_{D,t}^{I}}^{1-\theta} \left[\frac{z_{V,t}^{k-(\theta-1)} - {z_{D,t}^{I}}^{k-(\theta-1)}}{z_{V,t}^{k} - {z_{D,t}^{I}}^{k}} \right]^{-1} = \nu^{\theta-1} \left(\frac{z_{V,t}}{\tilde{z}_{D,t}^{I}} \right)^{\theta-1} = \frac{k}{k-(\theta-1)} \left(\frac{z_{V,t}}{\tilde{z}_{D,t}^{I}} \right)^{\theta-1}$$

E.2 The relationship between the average profit of domestic intangible firms and the average profit of domestic non-intangible firms

The average profit of Home firms that produce domestically without intangible investment is:

$$
\begin{aligned}
\tilde{d}^{NI}_{D,t} = d^{NI}_{D,t}(\tilde{z}^{NI}_{D,t}) &= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t\tilde{z}^{NI}_{D,t}}\right]^{1-\theta} C_t = \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right]^{1-\theta} C_t\tilde{z}^{NI\theta-1}_{D,t} \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right]^{1-\theta} C_t(\nu z_{min}z^{I}_{D,t})^{\theta-1}\left[\frac{z^{Ik-(\theta-1)}_{D,t} - z^{k-(\theta-1)}_{min}}{z^{Ik}_{D,t} - z^{k}_{min}}\right] \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t z^{I}_{D,t}}\right]^{1-\theta} C_t(\nu z_{min})^{\theta-1}\left[\frac{z^{Ik-(\theta-1)}_{D,t} - z^{k-(\theta-1)}_{min}}{z^{Ik}_{D,t} - z^{k}_{min}}\right] \\
&= d^{NI}_{D,t}(z^{I}_{D,t})(\nu z_{min})^{\theta-1}\left[\frac{z^{Ik-(\theta-1)}_{D,t} - z^{k-(\theta-1)}_{min}}{z^{Ik}_{D,t} - z^{k}_{min}}\right]
\end{aligned}
$$

The average profit of Home firms that produce domestically with intangible investment is:

$$
\begin{aligned}
\tilde{d}^{I}_{D,t} = d^{I}_{D,t}(\tilde{z}^{I}_{D,t}) &= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t\tilde{z}^{I}_{D,t}}\right]^{1-\theta} C_t - \phi_{D.t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t}\right]^{1-\theta} C_t\tilde{z}^{I\theta-1}_{D,t} - \phi_{D.t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t}\right]^{1-\theta} C_t(\nu z^{I}_{D,t}z_{V,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t} - z^{Ik-(\theta-1)}_{D,t}}{z^{k}_{V,t} - z^{Ik}_{D,t}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t z^{I}_{D,t}}\right]^{1-\theta} C_t(\nu z_{V,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t} - z^{Ik-(\theta-1)}_{D,t}}{z^{k}_{V,t} - z^{Ik}_{D,t}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= (\nu z_{V,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t} - z^{Ik-(\theta-1)}_{D,t}}{z^{k}_{V,t} - z^{Ik}_{D,t}}\right]\left\{\frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t z^{I}_{D,t}}\right]^{1-\theta} C_t - \phi_{D.t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)\right\} \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t} - z^{Ik-(\theta-1)}_{D,t}}{z^{k}_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= (\nu z_{V,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t} - z^{Ik-(\theta-1)}_{D,t}}{z^{k}_{V,t} - z^{Ik}_{D,t}}\right] d^{I}_{D,t}(z^{I}_{D,t}) \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t} - z^{Ik-(\theta-1)}_{D,t}}{z^{k}_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)
\end{aligned}
$$

The Home firm with productivity equal to the cutoff $z^I_{D,t}$ is indifferent between producing domestically with intangible investment or without it. I use the equality of profits at the productivity cutoff, $d^{NI}_{D,t}(z^I_{D,t}) = d^I_{D,t}(z^I_{D,t})$:

$$\begin{aligned}
\tilde{d}^I_{D,t} &= (\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] d^I_{D,t}(z^I_{D,t}) \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1) \\
&= (\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right]\left\{(\nu z_{min})^{1-\theta}\left[\frac{z_{D,t}^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\ k}_{D,t} - z^k_{min}}\right]^{-1} \tilde{d}^{NI}_{D,t}(z^I_{D,t})\right\} \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1) \\
&= (z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right](z_{min})^{1-\theta}\left[\frac{z_{D,t}^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\ k}_{D,t} - z^k_{min}}\right]^{-1} \tilde{d}^{NI}_{D,t}(z^I_{D,t}) \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1) \\
&= \frac{k}{k-(\theta-1)}\left(\frac{z^I_{D,t} z_{V,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right]\tilde{d}^{NI}_{D,t}(z^I_{D,t}) \\
&\quad + \left\{\frac{k}{k-(\theta-1)} z^{\theta-1}_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1)
\end{aligned}$$

The calculation process utilizes the domestic non-intangible firms' average productivity equation derived in Appendix D.

$$\tilde{z}^{NI}_{D,t} = \nu z_{min} z^I_{D,t}\left[\frac{z_{D,t}^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\ k}_{D,t} - z^k_{min}}\right]^{\frac{1}{\theta-1}}$$

$$\Rightarrow z^{1-\theta}_{min}\left[\frac{z_{D,t}^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\ k}_{D,t} - z^k_{min}}\right]^{-1} = \nu^{\theta-1}\left(\frac{z^I_{D,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1} = \frac{k}{k-(\theta-1)}\left(\frac{z^I_{D,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1}$$

Appendix F. Asymmetric Steady State

In this appendix, I derive the steady-state solution for the offshoring model incorporating intangible capital, accounting for cross-country asymmetry in the cost of effective labor (i.e., the terms of labor, $TOL < 1$).

To ascertain the unique steady-state solution, I obtain a numerical solution by resolving a system of 17 non-linear equations with 17 endogenous variables. The specific equations, numbered (36) through (52), are detailed herein. The variables in question represent the steady-state magnitudes of: z_V (the offshoring productivity cutoff in Home); z_H (the exporting productivity cutoff in Home); TOL (the terms of labor); $\frac{C}{C^*Q}$ (the real consumption ratio in units of the same consumption basket); Q (the real exchange rate); $\frac{d_D^I}{w}$, $\frac{d_D^{NI}}{w}$, $\frac{d_V}{w}$, $\frac{d_X}{w}$ (Home's real firm profits, specifically for domestic production with and without intangible investment, offshore production, and export market production, all divided by the real wage in Home); $\frac{d_D^{I*}}{w^*}$, $\frac{d_D^{NI*}}{w^*}$ (Foreign's real firm profits from domestic production, with and without intangible investment, divided by the real wage in Foreign); z_X^* (the exporting productivity cutoff in Foreign); ρ_X (the average exporting price in Home), ρ_X^*(the average exporting price in Foreign), $\frac{N}{N^*}$(the ratio of the number of firms between Home and Foreign). Once these 17 variables are numerically determined, I utilize them to derive the steady-state values of all remaining variables. For the purpose of calculating the steady-state solution, the following price and profit formulations are instrumental.

Using average profit and price in appendix C, this appendix computes the steady-state solution. Introducing $v = \frac{\beta(1-\delta)}{1-\beta(1-\delta)}d$, $N_E = \frac{\delta}{1-\delta}N$, and $v = f_E w$ into the Home firms' total profits, we can derive the first equation of the system.

$$\frac{1-\beta(1-\delta)}{\beta(1-\delta)}f_E = \frac{N_D^I}{N}\frac{\widetilde{d}_D^I}{w} + \frac{N_D^{NI}}{N}\frac{\widetilde{d}_D^{NI}}{w} + \frac{N_V}{N}\frac{\widetilde{d}_V}{w} + \frac{N_X}{N}\frac{\widetilde{d}_X}{w} \tag{36}$$

where $\frac{N_X}{N} = \left(\frac{1}{z_X}\right)^k$, $\frac{N_D^I}{N} = \left(\frac{1}{z_D^I}\right)^k - \left(\frac{1}{z_V}\right)^k$, $\frac{N_D^{NI}}{N} = 1 - \left(\frac{1}{z_D^I}\right)^k$, $\frac{N_V}{N} = \left(\frac{1}{z_V}\right)^k$.

Then, the profit formulas in the Home country are represented as follows:

$$\frac{\widetilde{d}_D^I}{w} = \frac{k}{k-(\theta-1)}f_X\frac{C}{C^*Q}Q^{1-\theta}\tau^{*(\theta-1)}\psi_D^{\xi(1-\theta)}\left(\frac{z_D^I z_V}{z_X}\right)^{\theta-1}\left[\frac{z_V^{k-(\theta-1)} - z_D^{I\ k-(\theta-1)}}{z_V^k - z_D^{I\ k}}\right] - \phi_D(\psi_D^{-\xi}-1) \tag{37}$$

$$\frac{\widetilde{d}_D^{NI}}{w} = \frac{k}{k-(\theta-1)}f_X\frac{C}{C^*Q}Q^{1-\theta}\tau^{*(\theta-1)}\left(\frac{z_{min}z_D^I}{z_X}\right)^{\theta-1}\left[\frac{z_D^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_D^{I\ k} - z_{min}^k}\right] \tag{38}$$

$$\frac{\widetilde{d}_V}{w} = \frac{k}{k-(\theta-1)}f_X\frac{C}{C^*Q}Q^{1-\theta}\left(\frac{\tau^*}{\tau}\right)^{(\theta-1)}\psi_V^{\xi(1-\theta)}TOL^{1-\theta}\left(\frac{z_V}{z_X}\right)^{\theta-1} - f_V TOL - \phi_V TOL(\psi_V^{-\xi}-1) \tag{39}$$

$$\frac{\widetilde{d}_X}{w} = \frac{\theta - 1}{k - (\theta - 1)} f_X \tag{40}$$

$$\frac{\widetilde{d}_V}{w} = {z_D^I}^{1-\theta}\left[\frac{z_V^{k-(\theta-1)} - {z_D^I}^{k-(\theta-1)}}{z_V^k - {z_D^I}^k}\right]^{-1}\left\{\frac{\widetilde{d}_D^I}{w} + \phi_D(\psi_D^{-\xi}-1)\right\} - \frac{k}{k-(\theta-1)}\phi_D(\psi_D^{-\xi}-1) + \frac{\theta-1}{k-(\theta-1)}TOL\Big\{f_V + \phi_V(\psi_V^{-\xi}-1)\Big\} \tag{41}$$

$$\frac{\widetilde{d}_D^I}{w} = \left(\frac{z_V}{z_{min}}\right)^{\theta-1}\left[\frac{z_V^{k-(\theta-1)} - {z_D^I}^{k-(\theta-1)}}{z_V^k - {z_D^I}^k}\right]\left[\frac{{z_D^I}^{k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{{z_D^I}^k - z_{min}^k}\right]^{-1}\frac{\widetilde{d}_D^{NI}}{w} + \left\{\frac{k}{k-(\theta-1)}z_V^{\theta-1}\left(\frac{z_V^{k-(\theta-1)} - {z_D^I}^{k-(\theta-1)}}{z_V^k - {z_D^I}^k}\right) - 1\right\}\phi_D(\psi_D^{-\xi}-1) \tag{42}$$

Then, the profit formulas in the Foreign country are represented as follows:

$$\frac{{\widetilde{d}_D^*}^I}{w^*} = \frac{k}{k-(\theta-1)}f_X^*\frac{C^*Q}{C}(\tau^*Q)^{(\theta-1)}\psi_D^{*\xi^*(1-\theta)}\left(\frac{z_D^{I*}}{z_X^*}\right)^{\theta-1} - \phi_D^*(\psi_D^{*-\xi^*}-1) \tag{43}$$

$$\frac{{\widetilde{d}_D^*}^{NI}}{w^*} = \frac{k}{k-(\theta-1)}f_X^*\frac{C^*Q}{C}(\tau^*Q)^{(\theta-1)}\left(\frac{z_{min}^* z_D^{I*}}{z_X^*}\right)^{\theta-1}\left[\frac{z_D^{I*\ k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{z_D^{I*\ k} - z_{min}^{*k}}\right] \tag{44}$$

$$\frac{{\widetilde{d}_D^*}^I}{w^*} = {z_{min}^*}^{1-\theta}\left[\frac{z_D^{I*\ k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{z_D^{I*\ k} - z_{min}^{*k}}\right]^{-1}\frac{\widetilde{d}_D^{NI*}}{w^*} - \frac{\theta-1}{k-(\theta-1)}\phi_D^*(\psi_D^{*-\xi^*}-1) \tag{45}$$

The total profit expression in the Foreign country is represented as follows:

$$\frac{1-\beta^*(1-\delta^*)}{\beta^*(1-\delta^*)}f_E^* = \frac{k}{k-(\theta-1)}f_X^*\frac{C^*Q}{C}Q^{\theta-1}\tau^{\theta-1}\left(\frac{z_{min}^* z_D^{I*}}{z_X^*}\right)^{\theta-1}\left[\frac{z_D^{I*\ k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{z_D^{I*k} - z_{min}^{*k}}\right](1 - z_D^{I*-k}) + \left\{\frac{k}{k-(\theta-1)}f_X^*\frac{C^*Q}{C}Q^{\theta-1}\tau^{\theta-1}\left(\frac{z_D^{I*}}{z_X^*}\right)^{\theta-1}\psi_D^{*\xi^*(1-\theta)} - \phi_D^*(\psi_D^{*\xi^*}-1)\right\}z_D^{I*-k} + \frac{\theta-1}{k-(\theta-1)}f_X^* z_X^{*-k} \tag{46}$$

The consumption ratio in units of the Home consumption basket is:

$$\frac{C}{C^*Q} = \frac{f_X^*}{f_X}TOL^\theta Q^{\theta-1}\left(\frac{\tau^* z_X}{\tau z_X^*}\right)^{\theta-1} \tag{47}$$

Under the condition of a balanced current account, it follows that:

$$z_V^{-k}TOL\left[\frac{k(\theta-1)}{k-(\theta-1)}\left(\frac{z_V}{z_X^*}\right)^{\theta-1}\psi_V^{\xi(1-\theta)}f_X^* - \Big(f_V + \phi_V(\psi_V^{-\xi}-1)\Big)\right] = \frac{k\theta}{k-(\theta-1)}\left\{\frac{1}{z_X^{*k}}\left(\frac{N}{N^*}\right)^{-1}f_X^*TOL - \frac{1}{z_X^k}f_X\right\} \tag{48}$$

The real exchange rate in the steady state is as follows:

$$Q^{1-\theta} = \left(\frac{N}{N^*}\right)^{-1} \frac{\Lambda}{\Omega} \tag{49}$$

$$\begin{aligned}\Lambda =&(1 - z_D^{I*-k}) TOL^{1-\theta} (z^*_{min} z_D^{I*})^{\theta-1} \left[\frac{z_D^{I*\ k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{z_D^{I*\ k} - z_{min}^{*k}}\right] + z_D^{I*-k} \psi_D^{*\xi^*(1-\theta)} \left(\frac{TOL}{z_D^{I*}}\right)^{1-\theta} \\ &+ z_X^{-k} \frac{N}{N^*} \left(\frac{\tau^*}{z_X}\right)^{1-\theta}\end{aligned}$$

$$\begin{aligned}\Omega =&(1 - z_D^{I\ -k})(z_{min} z_D^I)^{\theta-1} \left[\frac{z_D^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_D^{I\ k} - z_{min}^k}\right] \\ &+ (z_D^{I\ -k} - z_V^{-k}) \psi_D^{\xi(1-\theta)} (z_D^I z_V)^{\theta-1} \left[\frac{z_V^{k-(\theta-1)} - z_D^{I\ k-(\theta-1)}}{z_V^k - z_D^{I\ k}}\right] + z_V^{-k} \psi_V^{\xi(1-\theta)} \left(\frac{\tau TOL}{z_V}\right)^{1-\theta} \\ &+ z_X^{*-k} \left(\frac{N}{N^*}\right)^{-1} \left(\frac{\tau TOL}{z_X^*}\right)^{1-\theta}\end{aligned}$$

The equations for the export price are as follows:

$$\frac{\theta k}{k-(\theta-1)} f_X \frac{\widetilde{\rho}_X^{\theta-1}}{TOL} = 1 + \frac{1-\beta^*}{\beta^*(1-\delta^*)} f_E^* \frac{\widetilde{\rho}_X^{\theta-1}}{\Xi} \tag{50}$$

$$\frac{\theta k}{k-(\theta-1)} f_X^* TOL \widetilde{\rho}_X^{*\theta-1} = 1 + \frac{1-\beta}{\beta(1-\delta)} f_E \frac{\widetilde{\rho}_X^{*\theta-1}}{\Gamma} \tag{51}$$

$$\frac{N}{N^*} \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_X^*}\right)^{\theta-1} = \frac{\Xi}{\Gamma} \tag{52}$$

where

$$\begin{aligned}\Xi =&(1 - z_D^{*I\ -k}) \left(\frac{\tau^*}{TOL}\right)^{\theta-1} \left(\frac{z^*_{min} z_D^{I*}}{z_X}\right)^{\theta-1} \left[\frac{z_D^{I*\ k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{z_D^{I*\ k} - z_{min}^{*k}}\right] \\ &+ z_D^{*I\ -k} \left(\frac{\tau^*}{TOL}\right)^{\theta-1} \psi_D^{*-\xi^*(\theta-1)} \left(\frac{z_D^{I*}}{z_X}\right)^{\theta-1} + z_X^{-k} \frac{N}{N^*}\end{aligned}$$

$$
\begin{aligned}
\Gamma =& (1 - z_D^{I\ -k})(\tau TOL)^{\theta-1}\left(\frac{z_{min} z_D^I}{z_X^*}\right)^{\theta-1}\left[\frac{z_D^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_D^{I\ k} - z_{min}^k}\right] \\
&+ (z_D^{I\ -k} - z_V^{-k})(\tau TOL)^{\theta-1}\psi_D^{-\xi(\theta-1)}\left(\frac{z_D^I z_V}{z_X^*}\right)^{\theta-1}\left[\frac{z_V^{k-(\theta-1)} - z_D^{I\ k-(\theta-1)}}{z_V^k - z_D^{I\ k}}\right] \\
&+ z_V^{-k}\psi_V^{-\xi(\theta-1)}\left(\frac{z_V}{z_X^*}\right)^{\theta-1} + z_X^{*-k}\left(\frac{N}{N^*}\right)^{-1}
\end{aligned}
$$

Appendix G. The Real Exchange Rate and Log-linearization

The welfare consistent RER ($Q_t \equiv \epsilon P_t^*/P_t$) measures differences in a consumer's welfare derived from spending a given normal amount in each market, capturing pure variety effects. When the number of Home varieties increases more than that of Foreign, the welfare consistent RER (Q_t) depreciates and exceeds 1. In contrast, the data-consistent RER captures real price data, but it purges out the pure variety effect. As in GM05, the data-consistent RER ($\widetilde{Q}_t$) is related to the welfare-consistent RER (Q_t) by the equation:

$$\widetilde{Q}_t = \left(\frac{N_t^*}{N_t}\right)^{\frac{1}{\theta-1}} Q_t \tag{53}$$

where $N_t = N_{D,t}^{NI} + N_{D,t}^{I} + N_V + N_X^*$ denotes the total number of varieties available to Home consumers, and $N_t^* = N_{D,t}^{NI*} + N_{D,t}^{I*} + N_X$ represents the total number of products available to Foreign consumers.

To derive the data-consistent RER, I decompose welfare-based price indexes P_t and P_t^* into components representing (i) product variety and (ii) average prices as $P_t = (N_{D,t}^{I} + N_{D,t}^{NI} + N_{V,t} + N_{X,t}^*)^{\frac{1}{\theta-1}} \widetilde{P}_t$ for the Home country and $P_t^* = (N_{D,t}^{*I} + N_{D,t}^{*NI} + N_{X,t})^{\frac{1}{\theta-1}} \widetilde{P}_t^*$ for the Foreign country. The resulting expression for the data-consistent RER is:

$$\widetilde{Q}_{CPI,t}^{1-\theta} = \left(\frac{N_{D,t}^{I} + N_{D,t}^{NI} + N_{V,t} + N_{X,t}^*}{N_{D,t}^{*I} + N_{D,t}^{*NI} + N_{X,t}}\right) \frac{N_{D,t}^{*NI}\left(\frac{TOL_t}{\widetilde{z}_{D,t}^{*NI}}\right)^{1-\theta} + N_{D,t}^{*I}\left(\psi_{D,t}^{*\xi*}\frac{TOL_t}{\widetilde{z}_{D,t}^{*I}}\right)^{1-\theta} + N_{X,t}\left(\frac{\tau_t^*}{\widetilde{z}_{X,t}}\right)^{1-\theta}}{N_{D,t}^{NI}\left(\frac{1}{\widetilde{z}_{D,t}^{NI}}\right)^{1-\theta} + N_{D,t}^{I}\left(\frac{\psi_{D,t}^{\xi}}{\widetilde{z}_{D,t}^{I}}\right)^{1-\theta} + N_{V,t}\left(\frac{\psi_{V,t}^{\xi}\tau TOL_t}{\widetilde{z}_{D,t}^{I}}\right)^{1-\theta} + N_{X,t}^*\left(\frac{\tau TOL_t}{\widetilde{z}_{X,t}^*}\right)^{1-\theta}} \tag{54}$$

where the terms of labor $TOL_t = \frac{Qw_t^*/Z_t^*}{w_t/Z_t}$ measures the cost of effective labor in the Foreign country relative to that in the Home country; the iceberg trade costs τ_t and τ_t^* affect the imports of Home and Foreign, respectively. I log-linearize the data-consistent RER in equation (26) to derive economic implications. The detailed derivation process is provided in Appendix G, which also discusses the response channels of the data-consistent RER following the impulse responses.

Analytical results The log-linearized version of (54) is:

$$\begin{aligned}
\widetilde{\mathsf{Q}}_t = {} & (s_D - s_V + s_D^* - 1)\mathsf{TOL}_t && \text{(C1)}\\
& + s_V(\tilde{\mathsf{z}}_{V,t} - \mathsf{t}_t) + (s_D - s_D^I - s_V)\tilde{\mathsf{z}}_{D,t}^{NI} + s_D^I \tilde{\mathsf{z}}_{D,t}^{I} && \text{(C2,C3,C4)}\\
& + (1 - s_D)(\tilde{\mathsf{z}}_{X,t}^* - \mathsf{t}_t) - (1 - s_D^*)(\tilde{\mathsf{z}}_{X,t} - \mathsf{t}_t^*) - (s_D^* - s_D^{*I})\tilde{\mathsf{z}}_{D,t}^{*NI} - s_D^{*I}\tilde{\mathsf{z}}_{D,t}^{*I} && \text{(C5)}\\
& - \ s_V \xi \Psi_{V,t} - s_D^I \xi \Psi_{D,t} + s_D^{*I}\xi^* \Psi_{D,t}^* && \text{(C6)}\\
& + \frac{1}{\theta-1}\left(s_V - \frac{N_V}{N_D + N_V + N_X^*}\right)\left(\mathsf{N}_{V,t} - \mathsf{N}_{X,t}^*\right) && \text{(C7)}\\
& + \frac{1}{\theta-1}\left[\begin{array}{l}\left(\frac{N_D^{*NI}}{N_D^{*NI} + N_D^{*I} + N_X} - (s_D^* - s_D^{*I})\right)\left(\mathsf{N}_{D,t}^{*NI} - \mathsf{N}_{X,t}\right)\\ - \left(\frac{N_D^{NI}}{N_D^{NI} + N_D^{I} + N_V + N_X^*} - (s_D - s_D^I - s_V)\right)\left(\mathsf{N}_{D,t}^{NI} - \mathsf{N}_{X,t}^*\right)\end{array}\right] && \text{(C8)}
\end{aligned}$$

$$
+\frac{1}{\theta-1}\left[\begin{array}{l}\left(\dfrac{N_D^{*I}}{N_D^{*NI}+N_D^{*I}+N_X}-s_D^{*I}\right)\left(\mathsf{N}_{D,t}^{*I}-\mathsf{N}_{X,t}\right)\\ \quad-\left(\dfrac{N_D^{I}}{N_D^{NI}+N_D^{I}+N_V+N_X^{*}}-s_D^{I}\right)\left(\mathsf{N}_{D,t}^{I}-\mathsf{N}_{X,t}^{*}\right)\end{array}\right] \tag{C9}
$$

Variables denoted in sans-serif font represent percent deviations from their respective steady states. Parameter s_D represents the steady-state share of Home's spending on varieties produced by Home firms, both domestically and offshore. Within s_D, s_D^I is the share spent on varieties produced by the domestic intangible firms, and s_V on varieties produced by offshoring firms. The remainder, s_D^{NI}, is denoted $s_D - s_D^I - s_V$, representing spending on varieties produced by domestic firms without intangible investment. Similarly, s_D^* is the steady-state share of Foreign's spending on varieties produced by Foreign firms domestically, with s_D^{*I} and $(s_D^* - s_D^{*I})$ denoting the shares for intangible and non-intangible firms, respectively. The average productivities of Foreign intangible and non-intangible firms, $\tilde{z}_D^{*I}$ and $\tilde{z}_D^{*NI}$, remain constant over time.

The calibration ensures that: (a) $(s_D - s_V + s_D^*) > 1$, as domestically produced varieties represent more than half of the consumption expenditure in each country; (b) $\left(\frac{N_D^I}{N_D^{NI}+N_D^I+N_V+N_X^*} - s_D^I\right) > 0$ and $\left(\frac{N_D^{*I}}{N_D^{*NI}+N_D^{*I}+N_X} - s_D^{*I}\right) > 0$, indicating that less productive intangible-investing firms have smaller market shares than their share in the total number of varieties in both countries; (c) $\left(\frac{N_D^{NI}}{N_D^{NI}+N_D^I+N_V+N_X^*} - (s_D - s_D^I - s_V)\right) > 0$ and $\left(\frac{N_D^{*NI}}{N_D^{*NI}+N_D^{*I}+N_X} - (s_D^* - s_D^{*I})\right) > 0$, meaning that less productive non-intangible firms also have smaller market shares than their share in the total number of varieties in both countries; and (d) $\left(s_V - \frac{N_V}{N_D+N_V+N_X^*}\right) > 0$, showing that more productive offshoring firms hold larger market shares than their share in total varieties. As in Zlate16, the model captures that more productive offshoring firms dominate market shares compared to less productive domestic firms, consistent with the empirical findings of Kurz (2006).

Log linearization The data-consistent real exchange rate is $\widetilde{Q}_{CPI,t}^{1-\theta} = \frac{N_t{}^*}{N_t}\left(\frac{P_t^*\epsilon_t}{P_t}\right)^{1-\theta} = \frac{N_t^*}{N_t}Q_t^{1-\theta}$, where $N_t \equiv N_{D,t}^{NI} + N_{D,t}^{I} + N_{V,t} + N_{X,t}^*$ and $N_t{}^* \equiv N_{D,t}^{NI*} + N_{D,t}^{I*} + N_{X,t}$ represents the total number of available varieties for consumers in both Home and Foreign countries.

The real exchange rate is derived by using the equilibrium prices for each type of production in the respective countries.

$$
\begin{aligned}
\widetilde{Q}_{CPI,t}^{1-\theta} &= \frac{N_t}{N_t^*}\frac{N_{D,t}^{NI*}(\tilde{\rho}_{D,t}^{NI*}P_t^*\epsilon_t)^{1-\theta} + N_{D,t}^{I*}(\tilde{\rho}_{D,t}^{I*}P_t^*\epsilon_t)^{1-\theta} + N_{X,t}(\tilde{\rho}_{X,t}P_t^*\epsilon_t)^{1-\theta}}{N_{D,t}^{NI}(\tilde{\rho}_{D,t}^{NI}P_t)^{1-\theta} + N_{D,t}^{I}(\tilde{\rho}_{D,t}^{I}P_t)^{1-\theta} + N_{V,t}(\tilde{\rho}_{V,t}P_t)^{1-\theta} + N_{X,t}^*(\tilde{\rho}_{X,t}^*P_t)^{1-\theta}} \\
&= \frac{N_t}{N_t^*}\frac{N_{D,t}^{NI*}\left(\frac{w_t^*P_t^*\epsilon_t}{Z_t^*\tilde{z}_{D,t}^{NI*}}\right)^{1-\theta} + N_{D,t}^{I*}\left(\frac{\psi_{D,t}^{*\xi^*}w_t^*P_t^*\epsilon_t}{Z_t^*\tilde{z}_{D,t}^{I*}}\right)^{1-\theta} + N_{X,t}\left(\frac{\tau_t^*Q^{-1}w_tP_t^*}{Z_t\tilde{z}_{X,t}}\right)^{1-\theta}}{N_{D,t}^{NI}\left(\frac{w_tP_t}{Z_t\tilde{z}_{D,t}^{NI}}\right)^{1-\theta} + N_{D,t}^{I}\left(\frac{\psi_{D,t}^{\xi}w_tP_t}{Z_t\tilde{z}_{D,t}^{I}}\right)^{1-\theta} + N_{V,t}\left(\frac{\tau_t\psi_{V,t}^{\xi}Q_tw_t^*P_t}{Z_t^*\tilde{z}_{V,t}}\right)^{1-\theta} + N_{X,t}^*\left(\frac{\tau_tw_t^*P_tQ_t}{Z_t^*\tilde{z}_{X,t}^*}\right)^{1-\theta}}
\end{aligned}
$$

$$= \frac{\frac{N^{NI*}_{D,t}}{N^*_t}\left(\frac{TOL_t}{\widetilde{z}^{NI*}_{D,t}}\right)^{1-\theta} + \frac{N^{I*}_{D,t}}{N^*_t}\left(\frac{\psi^{*\xi^*}_{D,t}TOL_t}{\widetilde{z}^{I*}_{D,t}}\right)^{1-\theta} + \frac{N_{X,t}}{N^*_t}\left(\frac{\tau^*_t}{\widetilde{z}_{X,t}}\right)^{1-\theta}}{\frac{N^{NI}_{D,t}}{N_t}\left(\frac{1}{\widetilde{z}^{NI}_{D,t}}\right)^{1-\theta} + \frac{N^{I}_{D,t}}{N_t}\left(\frac{\psi^{\xi}_{D,t}}{\widetilde{z}^{I}_{D,t}}\right)^{1-\theta} + \frac{N_{V,t}}{N_t}\left(\frac{\psi^{\xi}_{V,t}\tau_t TOL_t}{\widetilde{z}_{V,t}}\right)^{1-\theta} + \frac{N^*_{X,t}}{N_t}\left(\frac{\tau_t TOL_t}{\widetilde{z}^*_{X,t}}\right)^{1-\theta}}$$

In the steady state, I assume $f_E = f^*_E, \tau = \tau^*$. I adhere to the notation used in Zlate (2010), $s_D \equiv N^{NI}_D\left(\frac{w}{Z\tilde{z}^{NI}_D}\right)^{1-\theta} + N^I_D\left(\psi^{\xi}_{D,t}\frac{w}{Z\tilde{z}^I_D}\right)^{1-\theta} + N_V\left(\tau TOL\psi^{\xi}_V\frac{w}{Z\tilde{z}_V}\right)^{1-\theta}$. Expression s_D represents the steady state share of Home spending on varieties produced by Home firms, both domestically and offshore. Expression $s_V \equiv N_V\left[\left(\frac{\tau TOL\psi^{\xi}_V}{\psi^{\xi}_D}\right)^{1-\alpha}\frac{w}{Zz_V}\right]^{1-\theta}$ denotes the steady-state share of Home spending on varieties produced by the offshoring Home firms only. (Note that $s_D - s_V > 0$) Expression s^*_D denotes the steady state share of Home spending on varieties produced by Home intangible firms. Expression $s^*_D \equiv N^*_D\left(Q\frac{\psi^{*}_D{}^{\xi^*}}{\psi^{\xi}_D}\frac{w^*}{Z\tilde{z}_D}\right)^{1-\theta}$ represents the steady-state share of foreign spending on varieties produced by Foreign domestic firms. Expression $s^{I*}_D \equiv N^*_D\left(Q\frac{\psi^{*}_D{}^{\xi^*}}{\psi^{\xi}_D}\frac{w^*}{Z\tilde{z}_D}\right)^{1-\theta}$ represents the steady-state share of foreign spending on varieties produced by Foreign intangible firms. The average productivity of foreign firms producing domestically, denoted by $\tilde{z}^*_D$, remains constant over time.

The log-linearized version of the data-consistent real exchange rate ($\widetilde{\mathsf{Q}}^{1-\theta}_t$) can be written as:

$$\widetilde{\mathsf{Q}}_t = \frac{\overline{NUM}dDEN_t - \overline{DEN}dNUM_t}{(\theta-1)\overline{NUMDEN}} = \frac{1}{\theta-1}\left[\frac{dDEN_t}{\overline{DEN}} - \frac{dNUM_t}{\overline{NUM}}\right]$$

Using above information, I derived the log-linearized version of the data-consistent real exchange rate($\widetilde{\mathsf{Q}}^{1-\theta}_t$):

$$\begin{aligned}
\widetilde{\mathsf{Q}}_t = \frac{1}{\theta-1}\Bigg[&(s_D - s^*_D - s_V)\left\{\widetilde{\mathsf{N}}^{NI}_{D,t} - \widetilde{\mathsf{N}}_t - (1-\theta)\tilde{\mathsf{z}}^{NI}_{D,t}\right\}\\
&+ s^I_D\left\{\widetilde{\mathsf{N}}^{I}_{D,t} - \widetilde{\mathsf{N}}_t + (1-\theta)(\xi\Psi_{D.t} - \tilde{\mathsf{z}}^{I}_{D,t})\right\}\\
&+ s_V\left\{\widetilde{\mathsf{N}}_{V,t} - \widetilde{\mathsf{N}}_t + (1-\theta)(\mathsf{t}_t + \mathsf{TOL}_t - \tilde{\mathsf{z}}_{V,t} - \xi\Psi_{V,t})\right\}\\
&+ (1-s_D)\left\{\widetilde{\mathsf{N}}^*_{X,t} - \widetilde{\mathsf{N}}_t + (1-\theta)(\mathsf{t}_t + \mathsf{TOL}_t - \tilde{\mathsf{z}}^*_{X,t})\right\}\\
&- (s^*_D - s^{I*}_D)\left\{\widetilde{\mathsf{N}}^{NI*}_{D,t} - \widetilde{\mathsf{N}}^*_t + (1-\theta)(\mathsf{TOL}_t - \tilde{\mathsf{z}}^{NI*}_{D,t})\right\}\\
&- s^{I*}_D\left\{\widetilde{\mathsf{N}}^{I*}_{D,t} - \widetilde{\mathsf{N}}^*_t + (1-\theta)(\xi^*\Psi^*_{D.t} + \mathsf{TOL}_t - \tilde{\mathsf{z}}^{I*}_{D,t})\right\}\\
&- (1-s^*_D)\left\{\widetilde{\mathsf{N}}_{X,t} - \widetilde{\mathsf{N}}^*_t + (1-\theta)(\mathsf{t}^*_t - \tilde{\mathsf{z}}_{X,t})\right\}\Bigg]
\end{aligned}$$

$$
\begin{aligned}
&= (s_D - s_V + s_D^* - 1)\mathsf{TOL}_t && \text{(C1)}\\
&\quad + s_V(\tilde{\mathsf{z}}_{V,t} - \mathsf{t}_t) + (s_D - s_D^I - s_V)\tilde{\mathsf{z}}_{D,t}^{NI} + s_D^I \tilde{\mathsf{z}}_{D,t}^I && \text{(C2, C3, C4)}\\
&\quad + (1 - s_D)(\tilde{\mathsf{z}}_{X,t}^* - \mathsf{t}_t) - (1 - s_D^*)(\tilde{\mathsf{z}}_{X,t} - \mathsf{t}_t^*) - (s_D^* - s_D^{*I})\tilde{\mathsf{z}}_{D,t}^{*NI} - s_D^{*I}\tilde{\mathsf{z}}_{D,t}^{*I} && \text{(C5)}\\
&\quad - s_V \xi \Psi_{V,t} - s_D^I \xi \Psi_{D,t} + s_D^{*I}\xi^* \Psi_{D,t}^* && \text{(C6)}\\
&\quad + \frac{1}{\theta - 1}\left(s_V - \frac{N_V}{N_D + N_V + N_X^*}\right)(\mathsf{N}_{V,t} - \mathsf{N}_{X,t}^*) && \text{(C7)}\\
&\quad + \frac{1}{\theta - 1}\left[\begin{array}{l}\left(\dfrac{N_D^{*NI}}{N_D^{*NI} + N_D^{*I} + N_X} - (s_D^* - s_D^{*I})\right)(\mathsf{N}_{D,t}^{*NI} - \mathsf{N}_{X,t})\\ \quad - \left(\dfrac{N_D^{NI}}{N_D^{NI} + N_D^I + N_V + N_X^*} - (s_D - s_D^I - s_V)\right)(\mathsf{N}_{D,t}^{NI} - \mathsf{N}_{X,t}^*)\end{array}\right] && \text{(C8)}\\
&\quad + \frac{1}{\theta - 1}\left[\begin{array}{l}\left(\dfrac{N_D^{*I}}{N_D^{*NI} + N_D^{*I} + N_X} - s_D^{*I}\right)(\mathsf{N}_{D,t}^{*I} - \mathsf{N}_{X,t})\\ \quad - \left(\dfrac{N_D^{I}}{N_D^{NI} + N_D^I + N_V + N_X^*} - s_D^I\right)(\mathsf{N}_{D,t}^{I} - \mathsf{N}_{X,t}^*)\end{array}\right] && \text{(C9)}
\end{aligned}
$$

Analytical results The log-linearized equation (26) shows nine channels (C1-C9) through intangible capital that affect the data-consistent RER: (1) price of non-traded varieties (via TOL_t); (2) offshore prices (via the average offshore productivity $\tilde{\mathsf{z}}_{V,t}$, and the magnitude of trade costs t_t); (3) price of domestic non-intangible varieties (through average non-intangible firm productivity $\tilde{\mathsf{z}}_{D,t}^{NI}$); (4) price of domestic intangible varieties (via the average productivity of intangible firms $\tilde{\mathsf{z}}_{D,t}^{I}$); (5) relative import prices (via the average productivity of Foreign exporters $(\tilde{\mathsf{z}}_{X,t}^* - \mathsf{t}_t)$ and that of Home exporters $(\tilde{\mathsf{z}}_{X,t} - \mathsf{t}_t^*)$; (6) marginal cost reductions $(- s_V \xi \Psi_{V,t} - s_D^I \xi \Psi_{D,t} + s_D^{*I}\xi^* \Psi_{D,t}^*)$; (7) relative difference in varieties between offshore varieties and imported varieties $(\mathsf{N}_{V,t} - \mathsf{N}_{X,t}^*)$; (8) relative difference in varieties between domestic non-intangible varieties and imported varieties $(\mathsf{N}_{D,t}^{NI} - \mathsf{N}_{X,t}^*)$; and (9) relative difference in varieties between domestic intangible varieties and imported varieties $(\mathsf{N}_{D,t}^{I} - \mathsf{N}_{X,t}^*)$.

Impulse responses Intangible investment amplifies data-consistent RER appreciation following a positive Home aggregate productivity shock (Lee (2025)). This is because it boosts domestic firm entry, lowering average productivity and raising domestic prices, leading to RER appreciation. The greater variety of Home goods further shifts consumer spending from Foreign to domestic, reinforcing this appreciation. Thus, intangibles strengthen the HBS effect, explaining the post-GFC RER appreciation alongside increased US intangible investment. This operates through channels C3, C4, C8, and C9. We explore the contributing channels, specifically the additional impact of intangible investment compared to Zlate (2010).

(C1) Price changes of non-traded varieties Without intangibles, a Home aggregate productivity increase encourages firm entry and lead to an appreciation of the terms of labor in the long run (i.e., TOL_t decreases). This results in a relative increase in the average price of non-traded varieties in Home, causing an appreciation of the data-consistent RER (i.e., $\widetilde{Q}_t^{CPI}$ decreases). Intangible investment can dampen this RER appreciation through two channels: (a) It leads to a dampened appreciation of TOL compared to Zlate (2010) (i.e., causes $\mathsf{TOL_t}$ to decrease by a smaller amount) because offshoring firms' intangible investments more significantly transfer wage pressures abroad. (b) Simultaneously, intangibles increase the impact of TOL on RER by lowering the spending share on offshore varieties and raising it on domestic varieties. This is seen in the coefficient of TOL_t in channel C1 $(s_D - s_V + s_D^* - 1)$, which increases as the offshore share (s_V) falls and the domestic share (s_D) rises. Without intangible capital, TOL appreciation decreases the domestic spending share (s_D). However, intangible capital dampens the TOL appreciation in the long run, preventing a sufficient decrease in the relative cost of hiring foreign labor for offshoring firms and mitigating upward pressure on domestic labor costs. This limits the expansion of offshoring firms, reducing the offshore spending share (s_V), and conversely, increases the domestic spending share (s_D). Consequently, intangible capital increases the coefficient $(s_D - s_V + s_D^* - 1)$, further contributing to the dampened TOL appreciation.

(C2) Price changes of offshore varieties This channel mirrors Zlate (2010). Rising Foreign wages reduce the number of offshoring firms but increase their average productivity. Intangible investment amplifies this average productivity increase $(\tilde{z}_{V,t})$, leading to a larger price drop in offshore varieties and dampening RER appreciation. However, long-run TOL appreciation increases offshoring profitability, growing the number of offshoring firms and decreasing their average productivity $\tilde{z}_{V,t}$, raising their average price over time and contributing to RER appreciation.

(C3 and C4) Price changes of domestically produced goods (without and with intangible capital) Intangible capital incorporation increases Home firms' profits and income, depreciating TOL. This initially raises the number of both non-intangible (N_D^{NI}) and intangible (N_D^{I}) firms, decreases their respective average productivities $(\tilde{z}_{D,t}^{NI}$ and $\tilde{z}_{D,t}^{I})$, and increases their average prices, leading to RER appreciation. However, as TOL appreciates over time, the increase in the number of both types of domestic firms slows, causing their average productivities to rise and average prices to fall, thus dampening the RER appreciation effect.

(C5) Relative import price changes Without intangible investment, offshoring increases Foreign labor demand, putting upward pressure on Foreign wages and weakening the export competitiveness of Foreign firms. As a result, the average productivity of surviving Foreign exporters $(\tilde{z}_{X,t}^*)$ rises relative to that of their Home counterparts $(\tilde{z}_{X,t})$, lowering the average price of imports in Home relative to Foreign and dampening RER appreciation. With intangible investment, this effect is amplified. Offshoring firms that invest in intangibles reduce marginal costs, further undermining the price competitiveness of Foreign firms. As a result, the average productivity of surviving Foreign exporters $(\tilde{z}_{X,t}^*)$ rises further relative to their Home counterparts $(\tilde{z}_{X,t})$, lowering the average price of imports in Home and further dampening RER appreciation.

(C6) Changes in the fraction of marginal cost reduction The incorporation of intangible investment reduces marginal costs, leading to lower production prices. When the price of Home's offshore-produced varieties falls by $\Psi_{V,t}$, it causes RER depreciation. Similarly, a price decline in domestically produced varieties by $\Psi_{D,t}$ due to intangible investment also contributes to RER depreciation. In contrast, a decline in the price of Foreign's domestically produced varieties by $\Psi^*_{D,t}$ leads to RER appreciation.

As discussed in (C1), intangible investment raises the spending share of domestically produced varieties with intangible investment in total spending (s_D^I) and reduces the share on offshore varieties (s_V). As a result, the depreciation effect on the RER caused by offshore varieties is diminished, while that from domestic intangible varieties becomes more pronounced.

(C7) Expenditure switching from imports towards offshore varieties Similar to the channel discussed in Zlate (2010), offshoring reduces Foreign export competitiveness. Since offshoring requires intangible investment, Foreign export competitiveness decreases even further. Consequently, Home consumers shift their expenditure from less competitive Foreign varieties to relatively cheaper offshore varieties, more significantly dampening RER appreciation.

(C8 and C9) Expenditure switching from imports towards domestic varieties (without and with intangible capital) When there is a positive aggregate productivity shock, the incorporation of intangibles generates higher profits for Home firms, resulting in a rise in Home income. Consequently, Home consumers spend more on Foreign-produced varieties, which increases Foreign labor demand and Foreign wages. This short-run TOL depreciation causes more domestic firms—both those investing in intangibles and those not—to enter the Home market. This firm entry in the Home country increases the number of domestic varieties ($\mathsf{N}_{D,t}^I$, $\mathsf{N}_{D,t}^{NI}$) compared to the number of imported foreign varieties ($\mathsf{N}_{X,t}^*$). Consequently, Home consumers shift their expenditure away from imported varieties toward domestically produced varieties, despite their relatively higher prices. This shift leads to an RER appreciation. However, as TOL appreciates in the long run, the number of domestically producing firms decreases. This reduction leads to a rise in average productivity and a decrease in average prices, easing the appreciation pressure on RER.